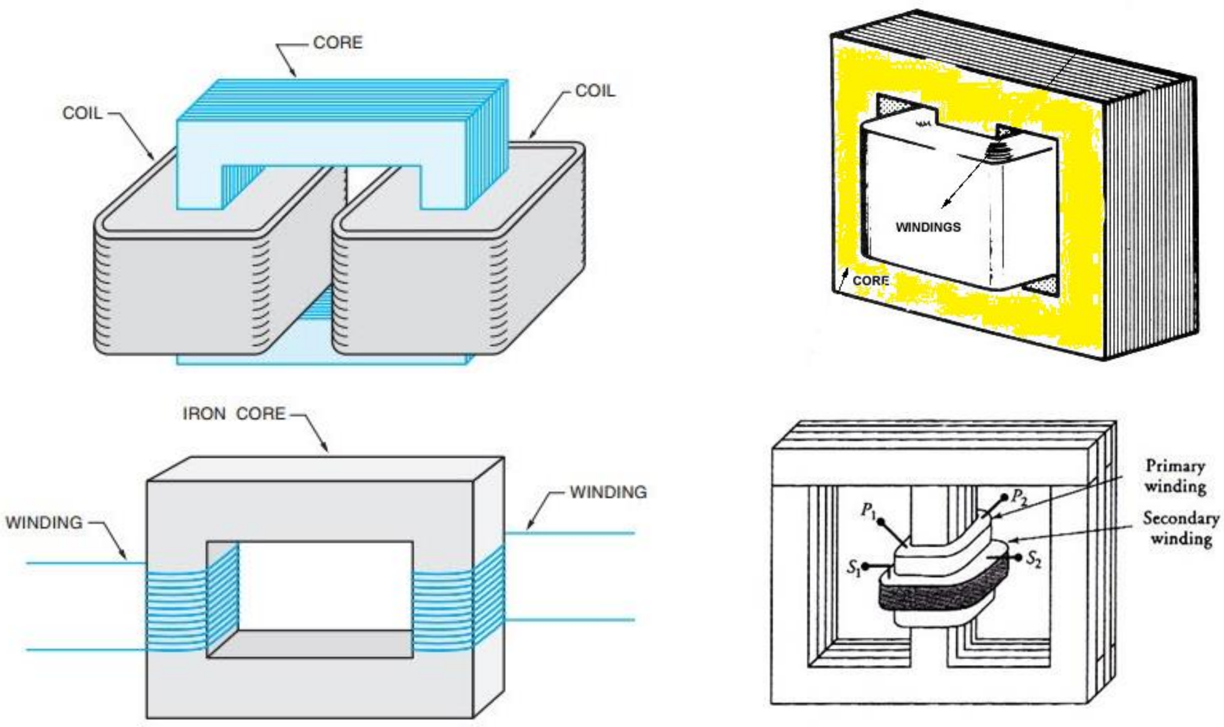
CHAPTER # 2 TRANSFORMERS

1. Introduction

The transformer can change the magnitude of alternating voltage or current from high to low values and vice versa. This useful property of transformer is mainly responsible for the widespread use of alternating currents rather than direct currents i.e., electric power is generated, transmitted and distributed in the form of alternating current. Transformers have no moving parts, rugged and durable in construction, thus requiring very little maintenance. They also have a very high efficiency—as high as 99%. In this chapter, we shall study some of the basic properties of transformers.

A transformer is a static machine. Although it is not an energy conversion device, it is essential in many energy conversion systems. It is a simple device, having two or more electric circuits coupled by a common magnetic field. Ferromagnetic cores are used to provide tight magnetic coupling and high flux densities. Such transformers are known as *iron-core transformers*. They are invariably used in high-power applications. *Air-core transformers* have poor magnetic coupling and are sometimes used in low power electronic applications.

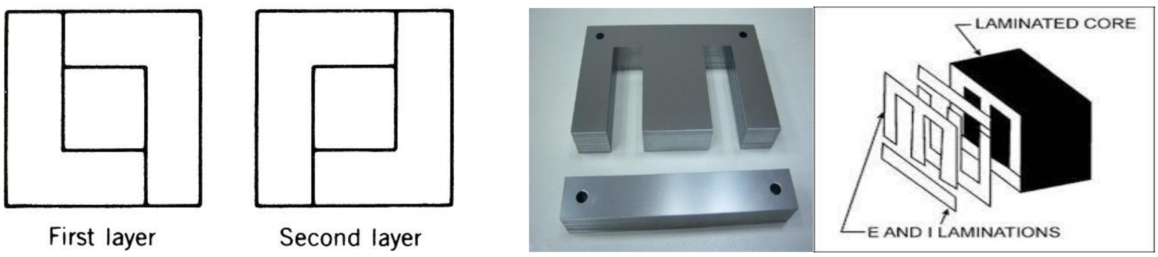
Two types of core constructions are normally used, as shown in Fig. 1. In the *core type* (Fig. 1a), the windings are wound around two legs of a magnetic core of rectangular shape. In the shell type (Fig. 1b), the windings are wound around the centre leg of a three-legged magnetic core.



a) Core type b) Shell type

Fig. 1 Core construction of single-phase transformer

To reduce core losses, the magnetic core is formed of a stack of thin laminations. Silicon-steel laminations of 0.014-inch thickness are commonly used for transformers operating at frequencies below a few hundred cycles. L-shaped laminations are used for core type construction (Fig. 2a) and E-I shaped laminations are used for shell-type construction (Fig. 2b).



a) L-shaped laminations b) E I-shaped laminations

Fig. 2 Core laminations of single-phase transformer

In case of single-phase transformers, there two windings. One winding is connected to an AC supply and referred as primary winding. The other winding is connected to an electrical load and referred as secondary winding. (See Fig. 3)

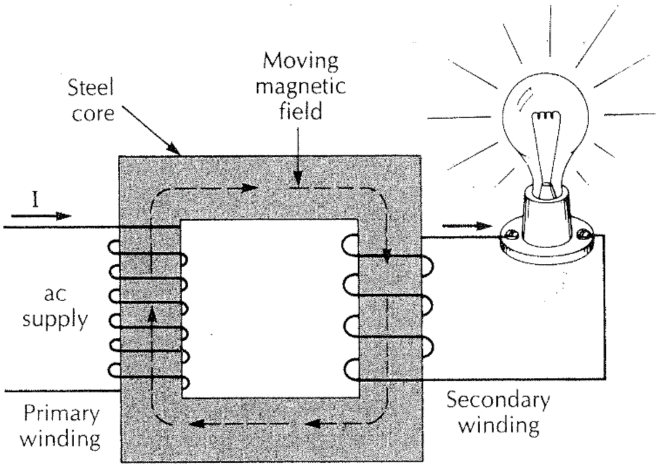


Fig. 3 Primary and secondary windings of 1-ph transformer

The winding with the higher number of turns will have a high voltage and is called the high-voltage (HV) or high tension (HT) winding. The winding with the lower number of turns is called the low-voltage (LV) or low-tension (LT) winding.

To achieve tighter magnetic coupling between the windings, they may be formed of coils placed one on top of another (Fig. 4a) or side by side (Fig. 4b)

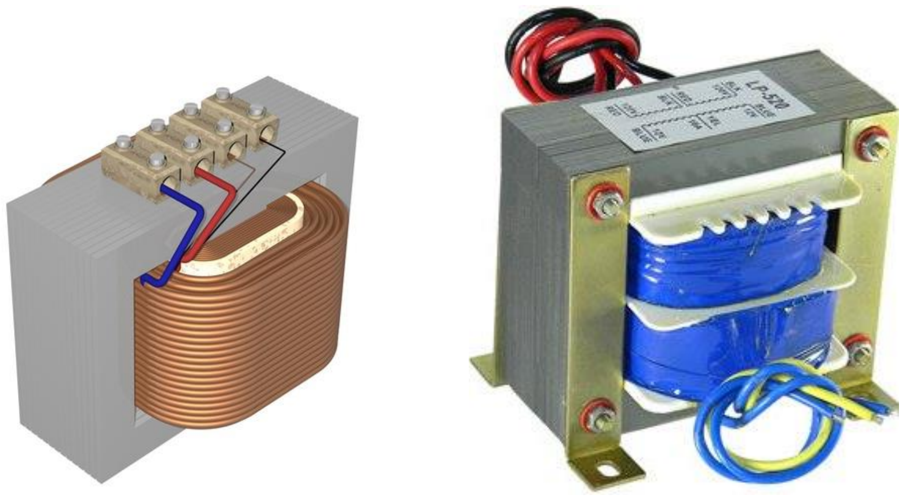


Fig. 4 a) Coils one on another b) Coils side by side

If there are more turns of wire on the primary than on the secondary, the output voltage will be lower than the input voltage. This is illustrated in Fig. 5 for a step-down and a step-up transformer. Notice that the winding with the greater number of turns has the higher voltage. In Fig. 5, one winding has twice as many turns as the other. In one case the voltage is stepped down to half, while in the other the voltage is stepped up to double.

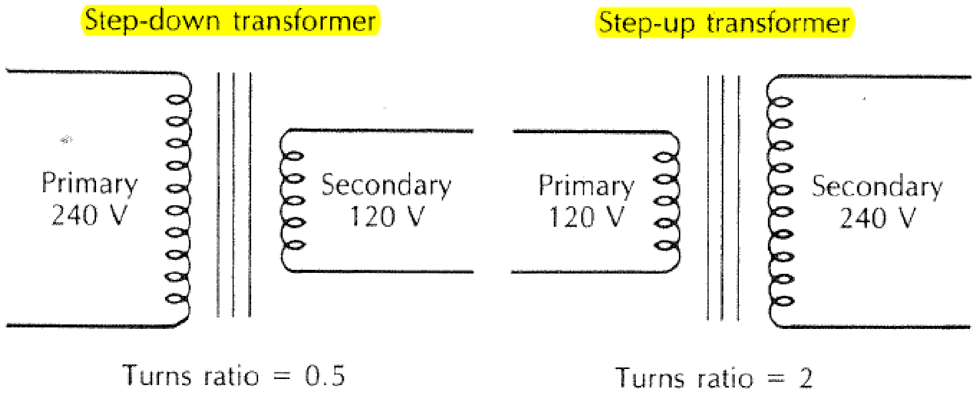
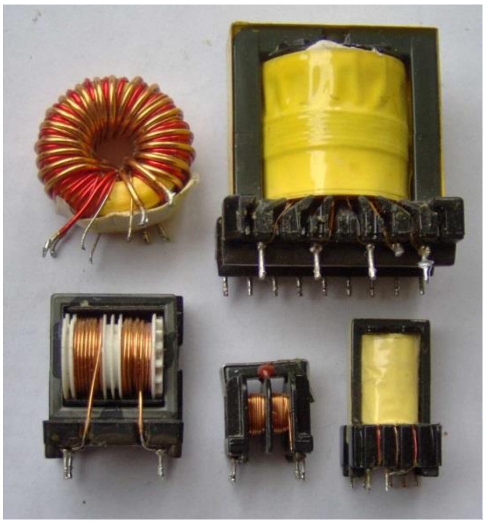


Fig. 5 Step- up and down transformers

It is important to know the ratio of the number of turns of wire on the primary winding as compared to the secondary winding. This is called the *turns ratio* of the transformer.

$$turns\ ratio\ (a) = \frac{number\ of\ secondary\ turns}{number\ of\ primary\ turns}$$

There are different shapes for small transformers as shown in Fig. 6.



single-phase Transformers

Three-phase transformer

Fig. 6 Different shape of transformers

Transformers have widespread use. Their primary function is to change voltage level. Electrical power is generated in a power station at about 30 kV. However, in domestic houses, electric power is used at 110 or 220 volts. Electric power is transmitted from a power plant to a load center at 200 to 500 kV. Transformers are used to step up and step down voltage at various stages of power transmission, as shown in Fig. 7.

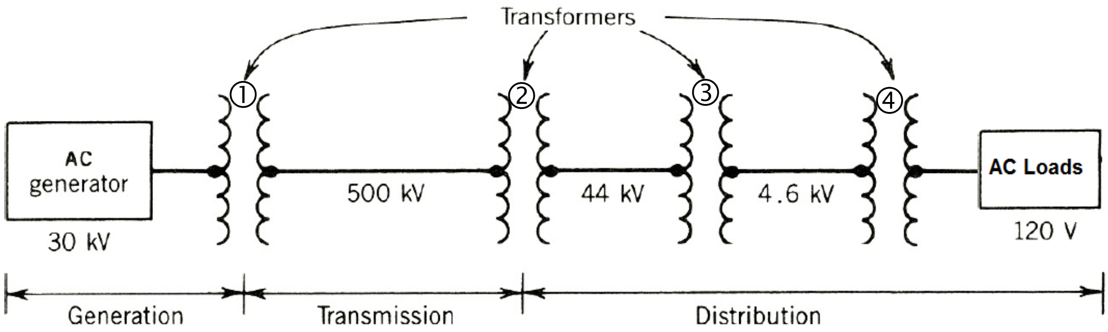


Fig. 7, Power transmission with step-up and down transformers

It is clear from the above figure that transformer #1 is step up transformer as it raised the voltage from 30 kV to 500 kV. But Transformers #2, #3 and #4 are step down transformers. Therefore, based on the primary and secondary voltages, transformers are classified to:

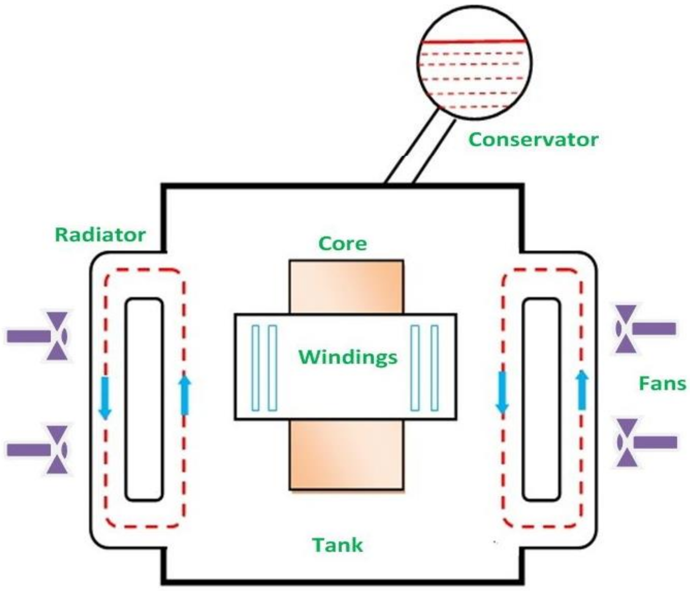
a) Step up transformers ($V_2 > V_1$) ($a > 1$)
b) Step down transformers ($V_2 < V_1$) ($a < 1$)
c) Isolating transformers ($V_2 = V_1$) ($a = 1$)

1.1 **Transformer Cooling**

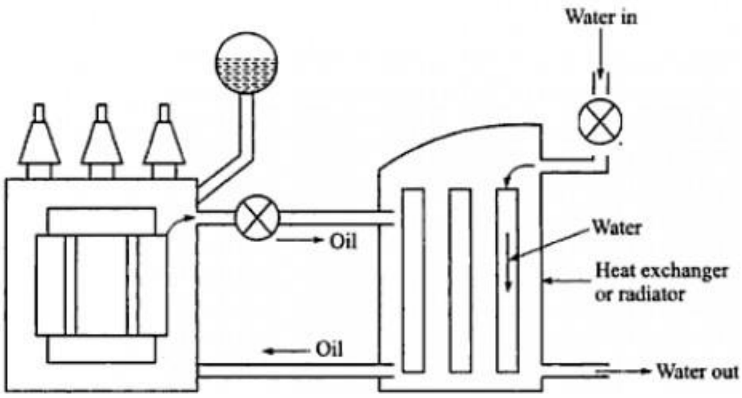
Another means of classifying the transformers is according to the type of cooling employed. The following types are in common use:

(a) air-blast type

(b) oil-filled self-cooled (Oil-filled transformers are built for outdoor duty)



(c) oil-filled water-cooled



1.2 <u>Transformer windings:</u>

Transformer windings are constructed of solid stranded copper or aluminum conductors, but copper is the most commonly used.

1.3 <u>Transformer windings insulation:</u>

The transformer windings are insulated by insulating material. The most important characteristic of the insulating material is its class. Class of insulation denotes the maximum temperature that it can withstand as shown in Fig.8. Classes A, E, B, F and H are used in dry-type transformers. For oil immersed transformers, class A is used.

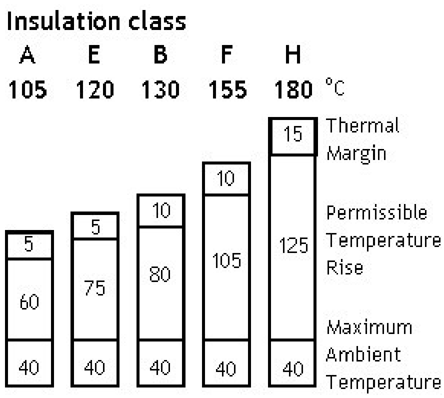


Fig. 8, Temperature limits according to IEC85 standard

1.4 Classification of transformers (based on their function):

- **Power transformers:** used in transmission lines and distribution systems. They have the highest power (VA) ratings compared to other types. The operating frequency is 50 – 60 Hz.
- **Electronic transformers:** used in electronic circuits. They are designed to operate over a wide range of frequencies (wide-band transformers) or over specific range of frequencies (narrow-band transformers).
- **Instrument transformers:** used to detect voltage or current in electronic circuits or in power systems. If they are used to detect voltage, they are called potential transformers. If they are used to detect current, they are called current transformers.
- **Audio and pulse transformers:** used in communication circuits.

1.5 Classification of transformers (based on number of windings):

- **Autotransformers:** have one winding with electrical connection.
- **Conventional transformers:** have 2 or more windings without electrical connection.

1.6 The following points may be noted carefully:

(i) The transformer action is based on the laws of electromagnetic induction.

(ii) There is no electrical connection between the primary and secondary. The AC power is transferred from primary to secondary through magnetic flux.

(iii) There is no change in frequency i.e., output power has the same frequency as the input power.

(iv) The losses that occur in a transformer are:

(a) core losses - eddy current and hysteresis losses

(b) copper losses - in the resistance of the windings

In practice, these losses are very small so that output power is nearly equal to the input primary power. In other words, a transformer has very high efficiency.

2. Ideal Transformer

Although ideal transformer cannot be physically realized, yet its study provides a very powerful tool in the analysis of a practical transformer. In fact, practical transformers have properties that approach very close to an ideal transformer.

Consider a transformer with two windings, a primary winding of N_1 turns and a secondary winding of N_2 turns, as shown schematically in Fig. 9. Ideal transformer has the following properties:

- The winding resistances are negligible.
- All fluxes are confined to the core and link both windings; that is, no leakage fluxes are present.
- Permeability of the core is infinite.
- The core losses (hysteresis & eddy) are negligible.

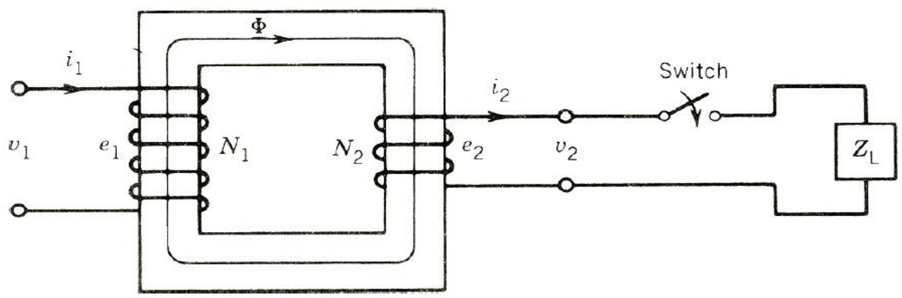


Fig. 9, Ideal transformer

When the primary winding is connected to a time-varying voltage v_1, a time-varying flux Φ is established in the core. A voltage e_1 will be induced in the winding and will equal the applied voltage if resistance of the winding is neglected:

$$v_1 = e_1 = N_1 \frac{d\Phi}{dt}$$

The core flux also links the secondary winding and induces a voltage e_2, which is the same as the terminal voltage v_2:

$$v_2 = e_2 = N_2 \frac{d\Phi}{dt}$$

From the above two equations:

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = a$$

where a is the turns ratio.

Let us now connect a load (by closing the switch in Fig. 9) to the secondary winding. A current i_2 will flow in the secondary winding, and the secondary winding will provide an mmf N_2i_2 for the core. This will immediately make a primary winding current i_1 flow so that a counter-mmf N_1i_1 can oppose N_2i_2.

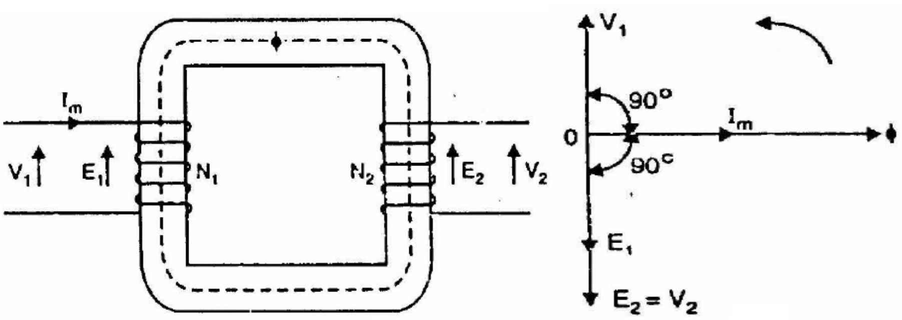
$$N_1 i_1 = N_2 i_2$$

$$\frac{I_2}{I_1} = \frac{N_1}{N_2} = \frac{1}{a}$$

$$\underset{\substack{\uparrow \\ \text{input volt-amperes}}}{V_1 I_1} = \underset{\substack{\uparrow \\ \text{output volt-amperes}}}{V_2 I_2}$$

2.1 <u>Phasor diagram of ideal transformer (at no load)</u>

Consider an ideal transformer on no load i.e., secondary is open-circuited as shown in Fig. (10 (i)). Under such conditions, the primary is simply a coil of pure inductance. When an alternating voltage V_1 is applied to the primary, it draws a small magnetizing current I_m which lags the applied voltage by 90°. This alternating current I_m produces an alternating flux ϕ which is proportional to and in phase with it. The alternating flux ϕ links both the windings and induces e.m.f. E_1 in the primary and e.m.f. E_2 in the secondary. The primary e.m.f. E_1 is, at every instant, equal to and in opposition to V_1 (Lenz's law). Both e.m.f.s E_1 and E_2 lag behind flux ϕ by 90°. However, their magnitudes depend upon the number of primary and secondary turns. Fig. (10 (ii)) shows the phasor diagram of an ideal transformer on no load. Since flux ϕ is common to both the windings, it has been taken as the reference phasor.



(i) Transformer circuit (ii) Phasor diagram

Fig. 10 Ideal transformer at no load

2.2 E.M.F. Equation of a transformer

Consider that an alternating voltage V_1 of frequency f is applied to the primary as shown in Fig. (10 (i)). The sinusoidal flux ϕ produced by the primary can be represented as:

$$\phi = \phi_m \sin\omega t$$

The instantaneous e.m.f. e_1 induced in the primary is

$$e_1 = -N_1 \frac{d\phi}{dt} = -N_1 \frac{d}{dt}(\phi_m \sin\omega t)$$
$$= -\omega N_1 \phi_m \cos\omega t = -2\pi f N_1 \phi_m \cos\omega t$$
$$= 2\pi f N_1 \phi_m \sin(\omega t - 90°)$$

The maximum value of induced e.m.f. in the primary is E_{m1}:

$$E_{m1} = 2\pi f N_1 \phi_m$$

The r.m.s. value E_1 of the primary e.m.f. is

$$E_1 = \frac{E_{m1}}{\sqrt{2}} = \frac{2\pi f N_1 \phi_m}{\sqrt{2}} = 4.44 f N_1 \phi_m$$

Similarly

$$E_2 = 4.44 f N_2 \phi_m$$

Example (1):

A single-phase transformer has 400 primary and 1000 secondary turns. The net cross-sectional area of the core is 60 cm^2. If the primary winding be connected to a 50-Hz supply at 520 V, calculate **(i)** the peak value of flux density in the core **(ii)** the voltage induced in the secondary winding.

$a = N_2/N_1 = 1000/400 = 2.5$

(*i*) $E_1 = 4.44\, f\, N_1\, B_m\, A$

or $520 = 4.44 \times 50 \times 400 \times B_m \times (60 \times 10^{-4})$ $\therefore$ $B_m =$ **0.976 Wb/m²**

(*ii*) $E_2/E_1 = a$ $\therefore$ $E_2 = a\,E_1 = 2.5 \times 520 =$ **1300 V**

Example (2):

A 25-kVA transformer has 500 turns on the primary and 50 turns on the secondary winding. The primary is connected to 3000-V, 50-Hz supply. Find the full-load primary and secondary currents, the secondary e.m.f. and the maximum flux in the core. Neglect leakage drops and no-load primary current.

$a = N_2/N_1 = 50/500 = 0.1$

Now, full-load $I_1 = 25{,}000/3000 =$ **8.33 A.**

F.L. $I_2 = I_1/a = 10 \times 8.33 =$ **83.3 A**

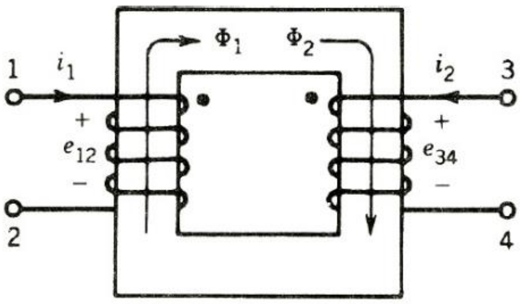
$E_2 = aE_1 = 3000 \times 0.1 =$ **300 *V***

Also, $E_1 = 4.44\, f\, N_1 \Phi_m$; $3000 = 4.44 \times 50 \times 500 \times \Phi_m$

$\therefore$ Φ_m **= 27 mWb**

2.2 Polarity of Transformer Windings

Windings of transformers or other electrical machines are marked to indicate terminals of like polarity. Consider the two windings shown in Fig. 11. Terminals 1 and 3 are identical, if the currents entering these terminals produce fluxes in the same direction in the core that forms the common magnetic path. For the same reason, terminals 2 and 4 are identical. If these two windings are linked by a common time-varying flux, voltages will be induced in these windings such that if at a particular instant the potential of terminal 1 is positive with respect to terminal 2, then at the same instant the potential of terminal 3 will be positive with respect to terminal 4.



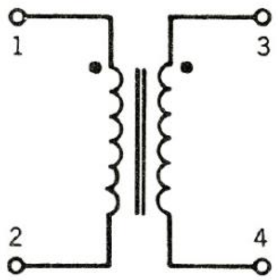
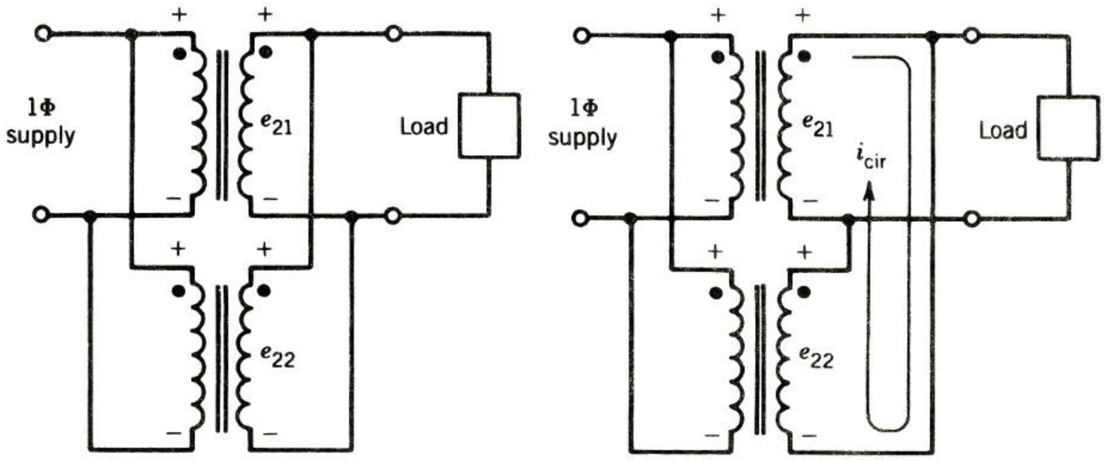


Fig. 11, Polarity of transformer windings

Polarities of windings must be known if transformers are connected in parallel to share a common load as shown in Fig. 12.



a) Correct connection b) Wrong connection

Fig. 12 Parallel connection of transformers

The relation between primary voltage and current with the secondary voltage and current is related by the dot notation as follows:

- If $\mathbf{V}_1$ and $\mathbf{V}_2$ are *both* either positive or negative at the dotted terminals, use positive turns ratio, otherwise use negative turns ratio.
- If I_1 and I_2 both enter into or both leave the dotted terminals, use negative turns ratio, otherwise use positive turns ratio.

Figure 13 explains the above rules:

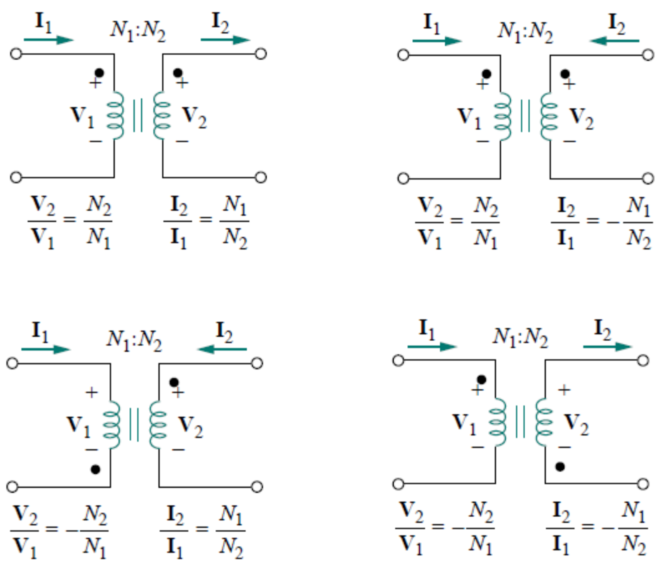


Fig. 13 Turns ratio sign

3. Shifting Impedances in a Transformer

Consider the transformer circuit shown in Fig. 14, then the ratio between the primary impedance Z_1 and the secondary impedance Z_2 that is Z_2/Z_1 is square of turns ratio as:

$$Z_2 = \frac{V_2}{I_2} \quad \text{and} \quad Z_1 = \frac{V_1}{I_1}$$

$$\therefore \ \frac{Z_2}{Z_1} = \left(\frac{V_2}{V_1}\right) \times \left(\frac{I_1}{I_2}\right) = a^2$$

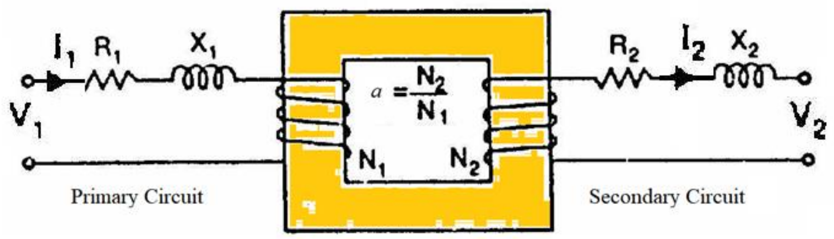


Fig. 14 Transformer Circuit

Note the importance of above relations. We can transfer the parameters from one winding to the other. Thus:

(i) A resistance R_1 in the primary becomes a^2 R_1 when transferred to the secondary.

(ii) A resistance R_2 in the secondary becomes R_2/a^2 when transferred to the primary.

(iii) A reactance X_1 in the primary becomes a^2 X_1 when transferred to the secondary.

(iv) A reactance X_2 in the secondary becomes $X_2/\ a^2$ when transferred to the primary.

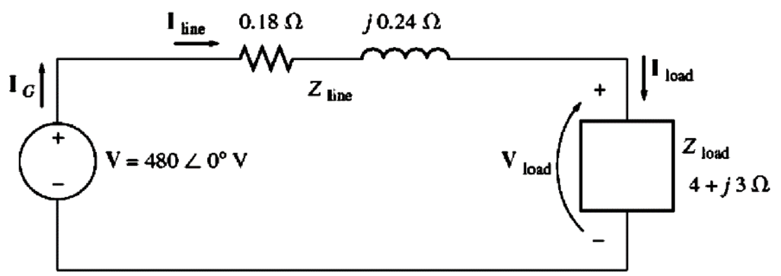
(v) When transferring voltage or current from one winding to another, only a is used.

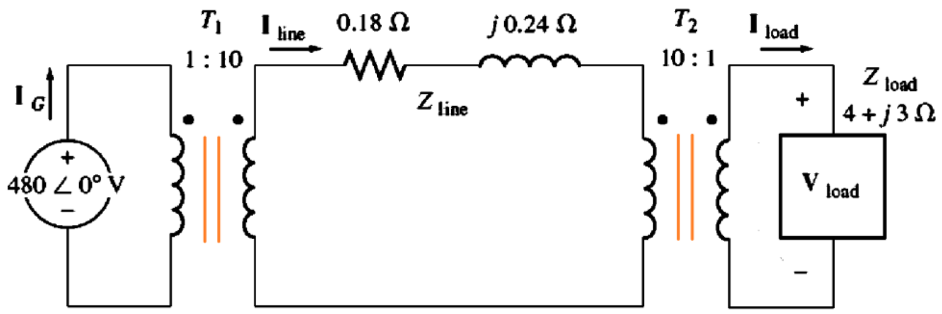
<u>Example (3):</u>

A single-phase power system consists of a 480-V, 60-Hz generator supplying a load $Z_{load} = 4 + j3.0\ \Omega$ through a transmission line of impedance $Z_{line} = 0.18 + j0.24\ \Omega$.

a) If no transformers are used, what will the voltage at the load be? What will the transmission line losses be?

b) If T_1 and T_2 transformers are used, what will the voltage at the load be? What will the transmission line losses be?





a) <u>If there no transformers are used:</u>

$$I_G = I_{line} = I_{load} = \frac{480}{(4+0.18)+j(3+0.24)} = 90.7601\angle-37.78\ A$$

$V_{load} = I_{load} \times Z_{load}$ = 90.7601∠-37.78 × (4+j3) = **453.8005∠-0.9101 V**

Losses at transmission line = I_{line}^2 (R_{line}) = $(90.7601)^2$ × 0.18 = **1482.7312 W**

b) <u>If the two transformers are used:</u>

Refer Z_{load} from the secondary of T_2 to the primary of T_2:

$$Z'_{load} = (4+j3)\times\left(\frac{10}{1}\right)^2 = 400 + j300\ ohm$$

Z_{load}' is connected in series with Z_{line}

Z_{tot} = 400.18 + 300.24 Ω

Refer Z_{tot} from the secondary of T_1 to the primary of T_1:

$$Z'_{tot} = (400.18+j300.24)\times\left(\frac{1}{10}\right)^2 = 4.0018 + j3.0024\ ohm$$

$$I_G = \frac{480}{(4.0018)+j(3.0024)} = 95.9447\angle-36.8795\ A$$

$$I_{line} = I_G \times \frac{1}{10} = 9.59447\angle-36.8795\ A$$

Losses at transmission line = I_{line}^2 (R_{line}) = $(9.59447)^2$ × 0.18 = **16.5698 W**

$$I_{load} = I_{line} \times \frac{10}{1} = 95.9447\angle - 36.8795\ A$$

$V_{load} = I_{load} \times Z_{load}$ = 95.9447∠-36.8795 × (4+j3) = **479.7235∠-0.0096 V**

Notice that raising the transmission voltage of the power system reduced transmission losses by 98.88%.

4. Practical Transformer

in a practical transformer the windings have resistances, not all windings link the same flux, permeability of the core material is not infinite, and core losses occur when the core material is subjected to time-varying flux. In the analysis of a practical transformer, all these imperfections must be considered.

A practical transformer differs from the ideal transformer in many respects. The practical transformer has

(i) iron losses
(ii) winding resistances and
(iii) magnetic leakage,

Iron losses: Since the iron core is subjected to alternating flux, there occurs eddy current and hysteresis loss in it. These two losses together are known as iron losses or core losses. The iron losses depend upon the supply frequency, maximum flux density in the core, volume of the core etc. It may be noted that magnitude of iron losses is quite small in a practical transformer.

Winding resistances: Since the windings consist of copper conductors, it immediately follows that both primary and secondary will have winding resistance. The primary resistance R_1 and secondary resistance R_2 act in series with the respective windings as shown in Fig. 10. When current flows through the windings, there will be power loss as well as a loss in voltage due to I×R drop. This will affect the power factor and E_1 will be less than V_1 while V_2 will be less than E_2.

Leakage reactances: Both primary and secondary currents produce flux. The flux ϕ_m which links both the windings is the useful flux and is called mutual flux (as shown in Fig. 14). However, primary current would produce some flux ϕ_{l1} which would not link the secondary winding. Similarly, secondary current would produce some flux ϕ_{l2} that would not link the primary winding. The flux such as ϕ_{l1} or ϕ_{l2} which links only one winding is called leakage flux. The leakage flux paths are mainly through the air. In other words, the effect of primary leakage flux ϕ_{l1} is to introduce an inductive reactance X_1 in series with the primary winding. Similarly, the secondary leakage flux ϕ_{l2} introduces an inductive reactance X_2 in series with the secondary winding. There will be no power loss due to leakage reactance. However, the presence of leakage reactance in the windings changes the power factor as well as there is voltage loss due to IX drop.

In a practical magnetic core having finite permeability, a magnetizing current I_m is required to establish a flux in the core. This effect can be represented by a magnetizing reactance X_m. Also, the core loss in the magnetic material can be represented by a resistance R_c.

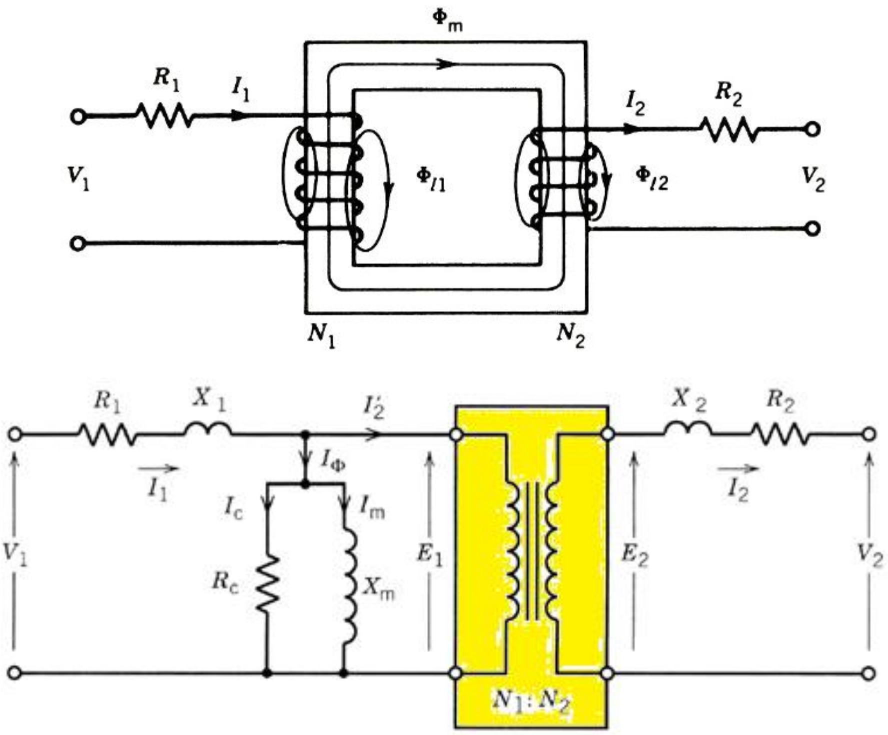


Fig. 14 Practical transformer

R_1 is the primary-winding resistance,

X_1 is the primary-winding leakage reactance,

R_2 is the secondary-winding resistance,

X_2 is the secondary-winding leakage reactance,

R_c is the core loss resistance,

X_m is the magnetizing reactance

The parallel branches R_c and X_m represent the no-load circuit. Also the phasor sum of both I_m and I_c gives the no-load current I_ϕ.

4.1 Exact Equivalent Circuits

The transformer circuit can be moved to the right or left by referring all quantities to the primary or secondary side, respectively. This is almost invariably done. The equivalent circuit moved to primary is shown in Fig. 15 below.

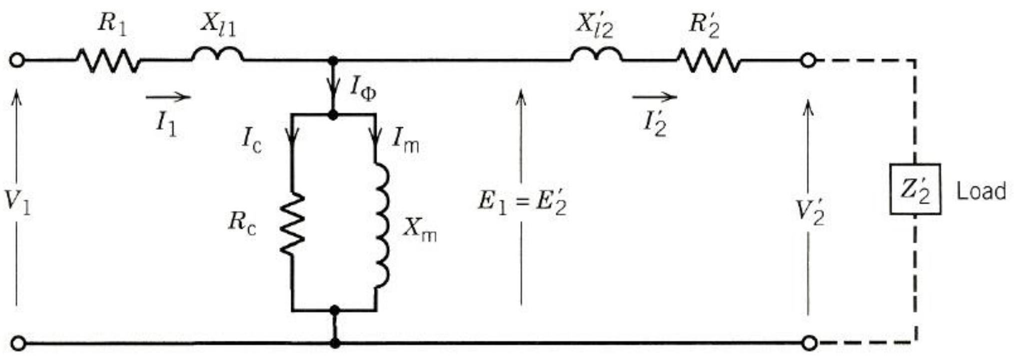


Fig. 15 Exact equivalent circuit referred to primary side

If we shift all the impedances from one winding to the other, the transformer core is eliminated and we get an equivalent electrical circuit. Various voltages and currents can be readily obtained by solving this electrical circuit.

4.2 Approximate Equivalent Circuits

The voltage drops I_1R_1 and I_1X_{l1} (Fig. 14) are normally small and $|E_1| = |V_1|$. If this is true, then the shunt branch (composed of R_c and X_m) can be moved to the supply terminal, as shown in Fig. 16. This approximate equivalent circuit simplifies computation of currents.

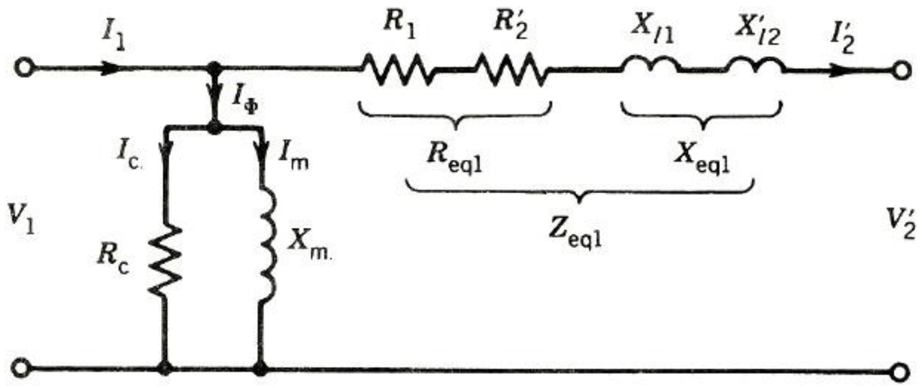


Fig. 16 Approximate equivalent circuit referred to primary side

4.3 More Approximate Equivalent Circuits

Generally, the exciting current I_Φ is a small percentage of the rated current of the transformer (less than 5%). A further approximation of the equivalent circuit can be made by removing the excitation branch, as shown in Fig. 17.

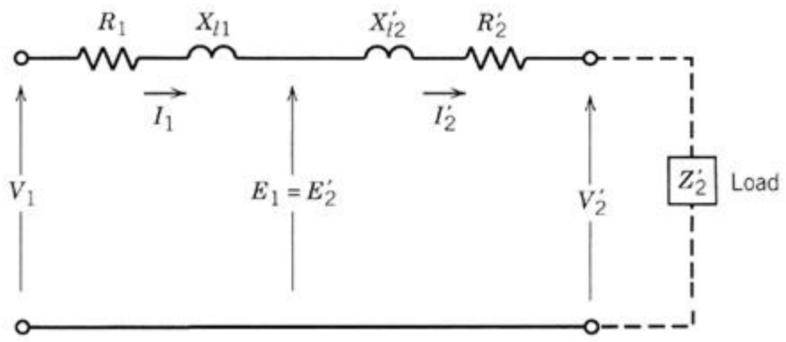


Fig. 17 More approximate equivalent circuit referred to primary side

4.4 Phasor Diagrams of Practical Transformer:

We shall consider two cases:

(i) when the winding resistance and leakage flux are neglected,

(ii) when the winding resistance and leakage flux are considered.

when the winding resistance and leakage flux are neglected

In case of transformer at no load, the phasor diagram is shown in Fig. 18:

- The flux (Φ) is considered as a reference on horizontal axis, and the induced voltage of the primary (E_1) and secondary (E_2) are lagging this flux by 90°

- The reactive component of current (I_m) is small in amount and in the same direction of the flux and lag the supply voltage (V_1) by 90°

$$I_m = I_\Phi \sin(\phi_o)$$

- The active component of current (I_c) which is in the same direction of the supply voltage (V_1)

$$I_c = I_\Phi \cos(\phi_o)$$

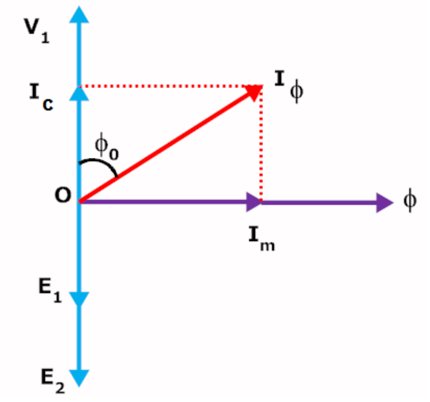


Fig. 18 Phasor diagram at no load

In case of inductive load which causes the secondary current I_2 to lag the secondary voltage V_2 by ϕ_2. The total primary current I_1 must meet two requirements: (a) It must supply the no-load current $I\phi$ to meet the iron losses in the transformer and to provide flux in the core. (b) It must supply a current I'_2 to counteract the demagnetizing effect of secondary currently I_2. The magnitude of I'_2 will be a I_2 and 180 out of phase as shown in Fig. 19.

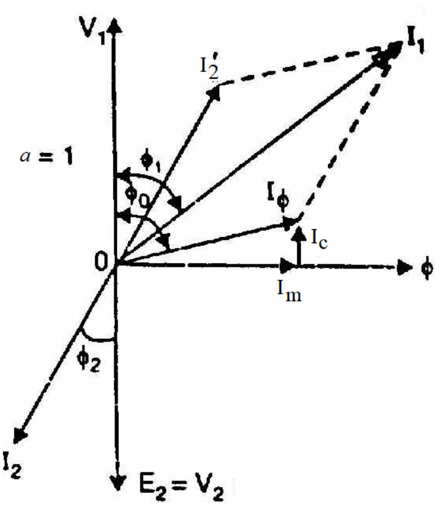


Fig. 19 Phasor diagram when R and X are neglected

Example:

(a) A 2,200/200-V transformer draws a no-load primary current of 0.6 A and absorbs 400 watts. Find the magnetising and iron loss currents. (neglect winding resistances and leakage reactances)

Iron-loss current I_c = 400/2200 = **0.182 A**

As we know, $I_\emptyset = \sqrt{I_c{}^2 + I_m{}^2}$ OR $I_m = \sqrt{I_\emptyset{}^2 - I_c{}^2}$

$$I_m = \sqrt{0.6^2 - 0.182^2} = \mathbf{0.572\ A}$$

(b) A 2,200/250-V transformer takes 0.5 A at a p.f. of 0.3 on open circuit. Find magnetising and working components of no-load primary current. (neglect winding resistances and leakage reactances)

$$\phi_o = cos^{-1}0.3 = 72.542$$

$$I_m = I_\Phi \sin(\phi_o) = 0.5 \times \sin(72.542) = \mathbf{0.477\ A}$$

$$I_c = I_\Phi \cos(\phi_o) = 0.5 \times 0.3 = \mathbf{0.15\ A}$$

Example:

A single-phase transformer has 1000 turns on the primary and 200 turns on the secondary. The no load current is 3 A, at a p.f. of 0.2 lag. Calculate the primary current and power-factor when the secondary current is 280 A at a p.f. of 0.80 lagging. (neglect winding resistances and leakage reactances)

Taking the primary voltage V_1 as a reference.

Turns ration (a) = 200/1000 = 0.2

Secondary power factor = 0.8 lag → ϕ_2 = - 36.87 (wrt V_2)

I_2 = 280 ∠-(180+36.87) A (wrt V_1)

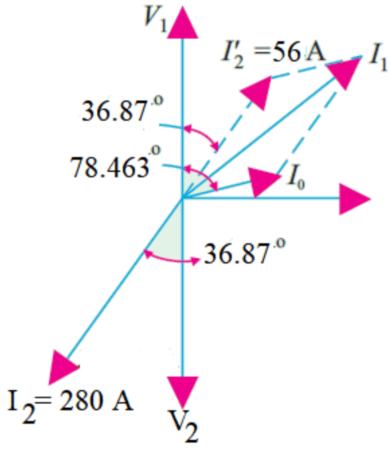
I_2' = 280 × 0.2 = 56 ∠-36.87 A

No-load power factor = 0.2 lag → ϕ_o = - 78.463 (wrt V_1)

I_ϕ = 3∠-78.463 A

I_1 = 56 ∠-36.87 + 3∠-78.463 = 58.278∠-38.828 A

Primary power factor = cos (38.828) = 0.779 lag



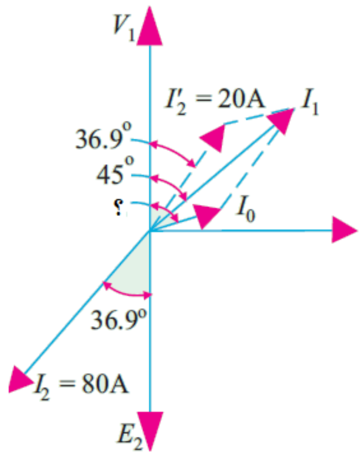
Example:

A transformer has a primary winding of 800 turns and a secondary winding of 200 turns. When the load current on the secondary is 80 A at 0.8 power factor lagging, the primary current is 25 A at 0.707 power factor lagging. Determine the no-load current of the transformer and its phase with respect to primary voltage.

Turns ration (a) = 200/800 = 0.25

$I_2' = 80 \times 0.25 = 20 \angle -36.9$ A

$I_\phi = 25\angle -45 - 20\angle -36.9 = 5.914\angle -73.457$ A



Example:

The primary of a certain transformer takes 1 A at a power factor of 0.4 when it is connected across a 200-V, 50-Hz supply and the secondary is on open circuit. The number of turns on the primary is twice that on the secondary. A load taking 50 A at a lagging power factor of 0.8 is now connected across the secondary. What is the value of primary current and its power factor?

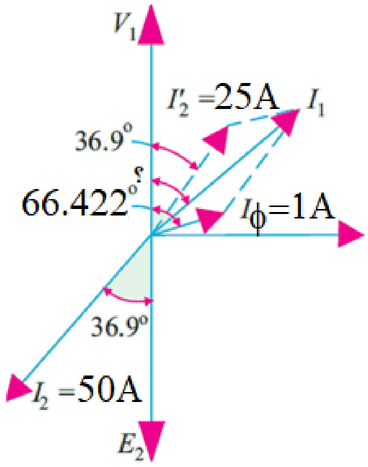
Turns ration (a) = $N_2 / N_1 = 0.5$

$I_2' = 50 \times 0.5 = 25 \angle -36.9$ A

$I_\phi = 1 \angle -66.422$

$I_1 = 25 \angle -36.9 + 1 \angle -66.422 = 25.8749 \angle -37.9912$ A

Primary power factor = cos (37.9912) = 0.7881 lag.



<u>when the winding resistance and leakage flux are considered</u>

This is the actual conditions that exist in a transformer. There is voltage drop in R_1 and X_1 so that primary e.m.f. E_1 is less than the applied voltage V_1. Similarly, there is voltage drop in R_2 and X_2 so that secondary terminal voltage V_2 is less than the secondary e.m.f. E_2.

Let us take the usual case of inductive load which causes the secondary current I_2 to lag behind the secondary voltage V_2 by ϕ_2. The total primary current I_1 will be the phasor sum of I'_2 and I_ϕ i.e.,

$$I_1 = I_\phi + I_2'$$

$$V_1 = -E_1 + I_1(R_1 + jX_1) = -E_1 + I_1 Z_1$$

$$V_2 = E_2 - I_2(R_2 + jX_2) = E_2 - I_2 Z_2$$

The above equations are represented by phasor diagram shown in Fig. 20.

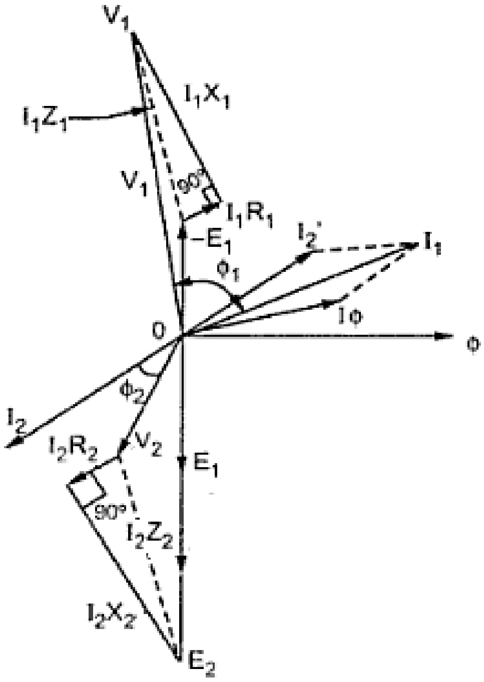


Fig. 20 Phasor diagram when R and X are considered (Inductive load)

Note that counter e.m.f. that opposes the applied voltage V_1 is $-E_1$. Therefore, if we add I_1R_1 (in phase with I_1) and $I_1 X_1$ (90° ahead of I_1) to $-E_1$, we get the applied primary voltage V_1. The phasor E_2 represents the induced e.m.f. in the secondary by the mutual flux ϕ. The secondary terminal voltage V_2 will be what is left over after subtracting I_2R_2 and I_2X_2 from E_2.

Now, we need to draw the phasor diagram at unity power factor. The only difference is that the angle (ϕ_2) between the load voltage V_2 and the load current I_2 become zero as shown in Fig. 21

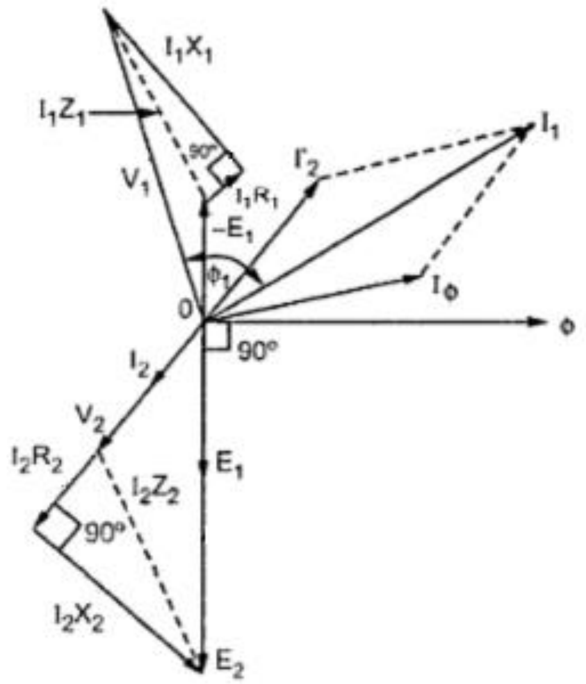


Fig. 21 Phasor diagram when R and X are considered (Resistive load)

If we need to draw the phasor diagram at leading power factor. The only difference is that the load current I_2 is leading the load voltage V_2 by angle (ϕ_2) as shown in Fig. 22

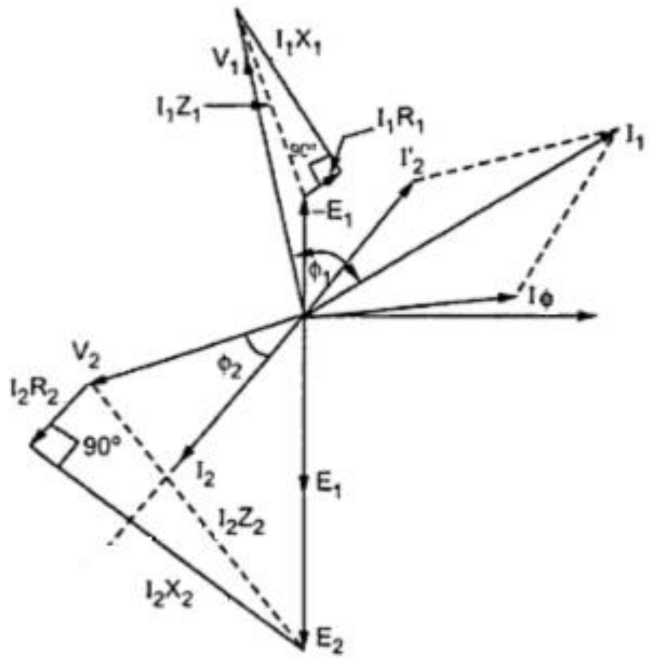


Fig. 22 Phasor diagram when R and X are considered (Capacitive load)

Example:

A 100 kVA, 1100/220 V, 50 Hz, single-phase transformer has a leakage impedance of (0.1 + J0.4) ohm for the H.V. winding and (0.006 + J0.015) ohm for the L.V. winding.

Find the equivalent winding resistance, reactance and impedance referred to the H.V. and L.V. sides.

Turns ratio = $(N_2/N_1) = (V_2/V_1)$ = 220/1100 = 0.2

(i) Referred to H.V. side (Primary):

Resistance = $r_1 + r_2'$ = 0.1 + [0.006 × (1100/220)^2] = 0.25 Ω

Reactance = $x_1 + x_2'$ = 0.4 + [0.015× (1100/220)^2] = 0.775 Ω

Impedance = $(0.25^2 + 0.775^2)^{0.5}$ = 0.8143 Ω

(ii) Referred to L.V. side (Secondary):

Resistance = $r_2 + r_1'$ = 0.006 + [0.1 × (0.2)^2] = 0.01 Ω

Reactance = $x_2 + x_1'$ = 0.015 + [0.4× (0.2)^2] = 0.031 Ω

Impedance = $(0.01^2 + 0.031^2)^{0.5}$ = 0.0328 Ω

Example:

The following data refer to a single-phase transformer:

Turn ratio $N_2:N_1$ = 1:19.5; R_1 = 25 Ω; X_1 = 100 Ω; R_2 = 0.06 Ω; X_2 = 0.25 Ω. No-load current = 1.25 A leading the flux by 30°. The secondary delivers 200 A at a terminal voltage of 500 V and p.f. of 0.8 lagging. Determine by the aid of a phasor diagram, the primary voltage, and primary p.f.

Assuming V_2 as reference vector, $V_2 = 500\angle 0$

$I_2 = 200\angle -36.87$ A

$E_2 = 500\angle 0 + 200\angle -36.87\ (0.06+j0.25) = 540.596\angle 3.479$ V

Now assuming the horizontal axis is taken as a reference vector,

$E_1 = 540.596\times 19.5 = 10541.622\ \angle 90$ V

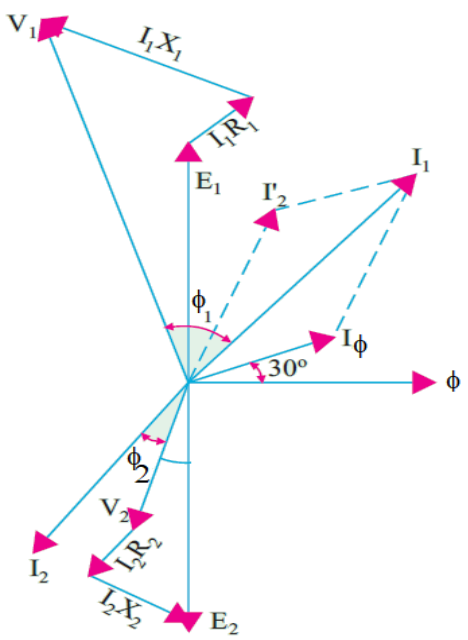
$I_2' = 200/19.5 = 10.2564\angle(180-90-36.87-3.479) = 10.2564\angle 49.651$ A

$I_1 = 1.25\angle 30 + 10.2564\angle 49.651 = 11.4413\angle 47.5454$ A

$V_1 = 10541.622\ \angle 90 + 11.4413\angle 47.5454\ (25 + j100) = \mathbf{11543.3325}\angle 93.2333$ V

The angle between V_1 and I_1 = 93.2333 – 47.5454 = 45.6879

Primary power factor = cos(45.6879) = **0.6986 lag**



Example:

The parameters of a 2300/230 V, 50-Hz transformer are given below:

$R_1 = 0.286\ \Omega$	$X_1 = 0.73\ \Omega$	$R_c = 250\ \Omega$
$R_2' = 0.319\ \Omega$	$X_2' = 0.73\ \Omega$	$X_m = 1250\ \Omega$

The load impedance $Z_L = 0.387 + j\ 0.29\ \Omega$.

Based on exact equivalent circuit with normal voltage across the primary, calculate:

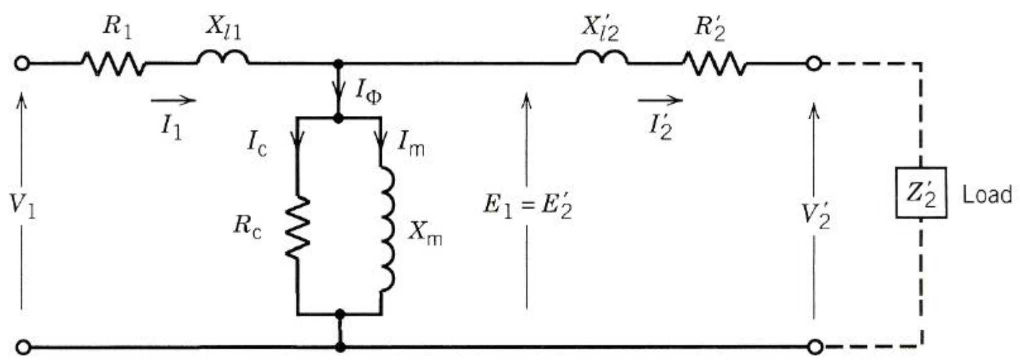
a) The primary current,

b) Primary power factor

c) The load current referred to primary,

d) The iron loss,

e) Efficiency.



$a = 230/2300 = 0.1$;

$Z_L = 0.387 + j\ 0.29\ \Omega$

$\mathbf{Z}_L' = \mathbf{Z}_L/a^2 = 100\ (0.387 + j\ 0.29) = 38.7 + j\ 29 = 48.36\ \angle 36.8462°\ \Omega$

$\therefore\ \mathbf{Z}_2' + \mathbf{Z}_L' = (38.7 + 0.319) + j(29 + 0.73) = 39.019 + j29.73 = 49.0546\ \angle\ 37.3051°\ \Omega$

$Y_m = (0.004 - j\ 0.0008)\ \Omega^{-1}$;

$\mathbf{Z}_m = 1/\mathbf{Y}_m = 240 + j48 = 245.1452\ \angle 11.3099°\ \Omega$

$\mathbf{Z}_m\ //\ (\mathbf{Z}_2' + \mathbf{Z}_L') = 245.1452\ \angle 11.3099°\ //\ 49.0546\ \angle\ 37.3051° = 41.4622\ \angle 33.0538°$

$\mathbf{Z}_{tot} = \mathbf{Z}_1 + \mathbf{Z}_m\ //\ (\mathbf{Z}_2' + \mathbf{Z}_L') = (0.286+j\ 0.73) + 41.4622\ \angle 33.0538° = 42.1026\angle 33.6742°$

Assuming that primary voltage is a reference vector,

a) $I_1 = 2300\angle 0\ /\ 42.1026\angle 33.6742° =$ **$54.6285\angle -33.6742°$ A**

b) Primary power factor = cos(33.6742) = **0.8322 lag**

$\mathbf{Z}_m + (\mathbf{Z}_2' + \mathbf{Z}_L') = 245.1452\ \angle 11.3099° + 49.0546\ \angle\ 37.3051° = 290.035\ \angle 15.5612°$

$$c)\ I_2' = I_1 \times \frac{Z_m}{Z_m + z_2' + Z_L} = \mathbf{46.1734\angle - 37.9255\ A}$$

$$I_\emptyset = I_1 - I_2' = 9.2396\angle - 11.9302\ A$$

$$I_c = I_\emptyset \times \frac{jX_m}{R_c + jX_m} = 9.0601\angle - 0.6203\ A$$

d) Iron Loss (P_{iron}) = $I_c^2 \times R_c$ = **20521.353 W**

Primary Cupper Loss (P_{cu1}) = $I_1^2 \times R_1$ = 853.5021 W

Secondary Cupper Loss (P_{cu2}) = $\mathbf{I}_2'^2 \times R_2'$ = 680.1025 W

Output power (P_{out}) = $\mathbf{I}_2'^2 \times R_L'$ = 82507.737 W

Input Power (P_{in}) = $P_{out} + P_{Loss}$ = 104562.6946 W

e) Efficiency (η) = P_{out} / P_{in} = **78.9074%**

Example:

A single-phase, 300 kVA, 11000V/2200V, 60 Hz transformer has the following equivalent circuit parameters referred to HV side:

$R_C = 57.6$ kΩ, $X_m = 16.34$ kΩ, $R_{eq} = 2.784$ Ω, $X_{eq} = 8.45$ Ω.

Load impedance referred to LV side is $16\angle 60°$ Ω,

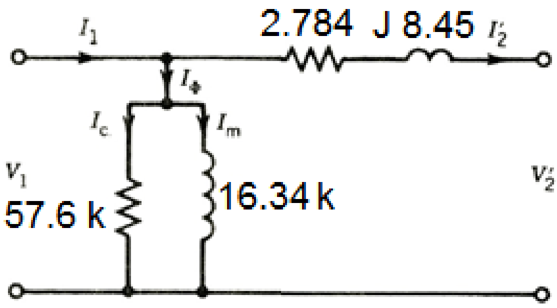
Based on approximate equivalent circuit, calculate:

a) No-load current as a percentage of full-load current, and no-load power factor,

b) No-load power (iron loss),

c) Voltage regulation (ε),

d) Efficiency (η).



At no load:

$Z_m = R_C // X_m = \frac{57600 \times j16340}{57600 + j16340} = 15719.71832\angle 74.1624°$

$$I_\varphi = \frac{V_1}{Z_m} = \frac{11000}{15719.71832\angle 74.1624°} = 0.6998\angle -74.1624°\ A$$

The full-load primary current I_1 = 300000/11000 = 27.2727 A

I_Φ as a percentage = 0.6998 / 27.2727 = 2.566 % ### a)

No-load power factor = cos (74.1624) 0.2729 lag. ### a)

The no-load loss (Iron loss) = $(V_1)^2/R_c$ = $(11000)^2/57600$ = 2100.6944 W ### b)

Now the load is connected and referred to primary side

$$Z_L' = 16 \times \left(\frac{11000}{2200}\right)^2 = 400\angle 60 = 200 + j346.4102\ \Omega$$

$$Z_{eq}' = (200 + 2.784) + j(346.4102 + 8.45) = 202.784 + j354.8602\ \Omega$$

$$I_2' = \frac{11000}{202.784 + j354.8602} = 26.9137\angle -60.2543°\ A$$

$$V_2' = I_2' \times Z_L' = 10765.47484\angle -0.2543°$$

$$\epsilon = \frac{V_1 - V_2'}{V_2'} = \frac{11000 - 10765.47484}{10765.47484} \times 100\% = 2.1785\ \%$$

Output power = $(I_2')^2 \times R_L = (26.9137)^2 \times 200 = 144869.4495$ W

Copper loss = $(I_2')^2 \times R_{eq} = (26.9137)^2 \times 8.45 = 6120.7342$ W

$$\eta = \frac{144869.4495}{144869.4495 + 6120.7342 + 2100.6944} = 94.6297\%$$

Another way to calculate voltage regulation

$$V_2' = 2200 \times \frac{11000}{2200} = 11000\ V$$

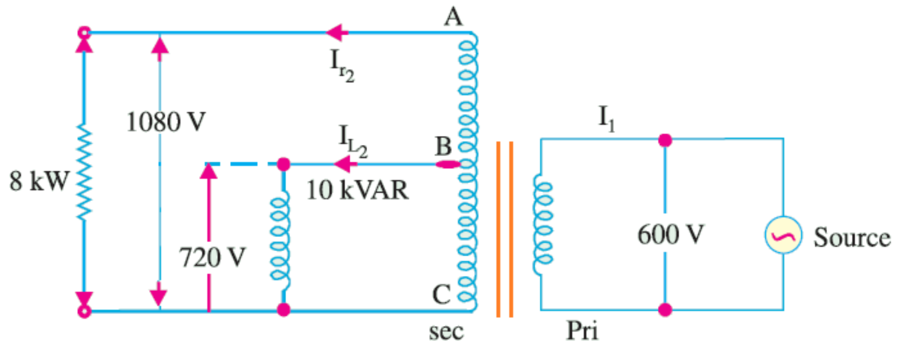
$$I_2' = \frac{11000}{400\angle 60} = 27.5\angle -60\ A$$

$$V_1 = 11000 + 27.5\angle -60\ (2.784 + j8.45) = 11239.6334$$

$$\epsilon = \frac{11239.6334 - 11000}{11000} = 2.1785\ \%$$

Example:

A transformer has a primary winding with a voltage-rating of 600 V. Its secondary-voltage rating is 1080 V with an additional tap at 720 V. An 8 kW resistive load is connected across 1080-V output terminals. A purely inductive load of 10kVAR is connected across the tapping point and common secondary terminal to get 720 V. Calculate the primary current and its power-factor.



Loads are connected as shown above;

I_{r2} = 8000/1080 = 7.4074 A at unity p.f. (resistive load)

I_{L2} = 10000 / 720 = 13.8889 A at zero lagging p.f. (pure inductive load)

These currents are reflected on to the primary sides with appropriate ratios of turns, with corresponding power factors.

I'_{r2} = 7.4074 × 1080/600 = 13.3333 A at unity p.f.

I'_{L2} = 13.8889 × 720/600 = 16.6667 A at zero lag. p.f.

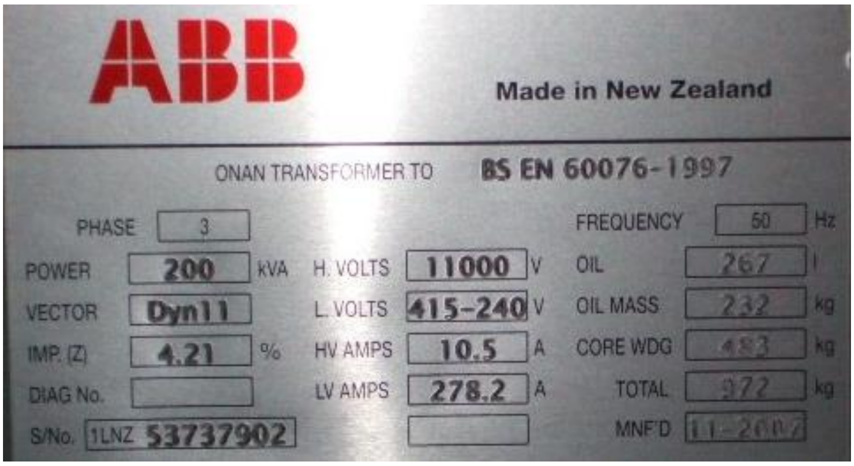
Hence, I_1= 13.3333∠0 + 16.6667∠-90 = 21.3438∠-51.3403 A,

Primary power factor is cos(51.3403) = 0.6247 lag.

5. Transformer Rating

There are many items must be appeared on the transformer nameplate: e.g.

- Name of manufacturer
- Serial number
- Year of manufacture
- Number of phases
- kVA or MVA rating
- Frequency
- Voltage ratings.
- Insulating class
- Approximate weight of the transformer
- Instruction for Installation and Operation



For example, a typical transformer may carry the following information on the nameplate: 10 kVA, 1100/110 volts. This means, the transformer has two windings, one rated for 1100 volts and the other for 110 volts. These voltages are proportional to their respective numbers of turns, and therefore the voltage ratio also represents the turns ratio (a=1100/110=10).

The 10 kVA rating means that each winding is designed for 10 kVA. Therefore, the current rating for HV winding is 10000/1100=9.09 A and for the LV winding is

10000/110=90.9 A. Actually, the winding that is connected to the supply will carry an additional component of current (excitation current), which is very small compared to the rated current of the winding.

6. The Per-Unit System of Measurements

As the size of a machine or transformer varies, its internal impedances vary widely. Thus, a primary circuit reactance of 0.1 Ω might be a high number for one transformer and a low number for another; it depends on the device's voltage and power ratings. However by using the per-unit system, impedance fall within fairly narrow ranges for each type and construction of the device. In the per-unit system, the voltages, currents, powers, impedances, and other electrical quantities are not measured in their usual SI units (volts, amperes, watts, ohms, etc.). Instead, each electrical quantity is measured as a decimal fraction of some base level as:

$$Quantity\ in\ per\ unit = \frac{actual\ value}{base\ value}$$

For example, if we have Z=10+j20 Ω (actual value) and the base impedance is 100 Ω, then the P.U. value of this impedance is calculated as:

$$Z_{pu} = \frac{10 + j20}{100} = 0.1 + j0.2 \qquad P.U.$$

Example:

A simple power system shown below, this system contains 480-V generator connected to a load via transmission line and two transformers. The base values for this system are

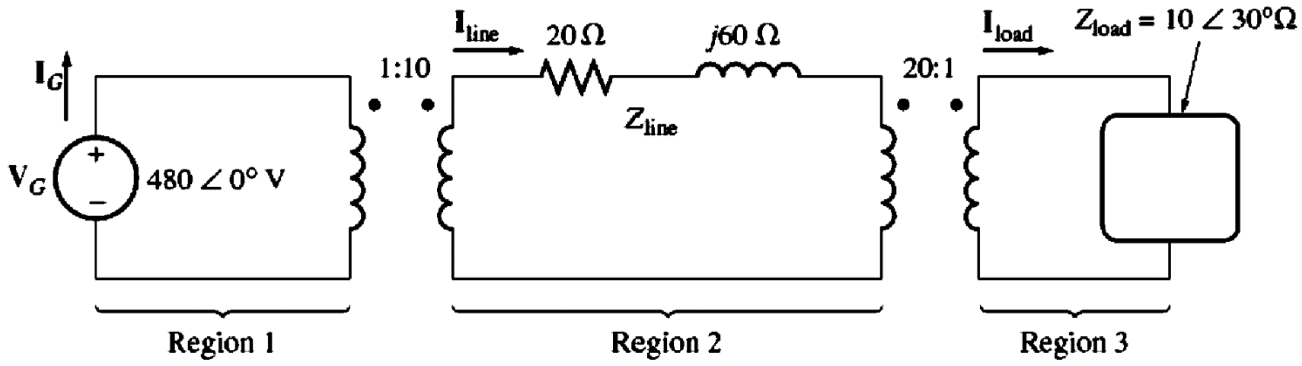
chosen to be 480 V and 10 kVA at the generator.

(a) Find the base voltage, current, impedance, and apparent power at each region.

(b) Convert this system to its per-unit equivalent circuit.

(c) Find the voltage, active and reactive power supplied to the load in this system.

(d) Find the power lost in the transmission line.



For region #1,	For region #2,	For region #3,
V_{base1} = 480 V, S_{base1} = 10 kVA, I_{base1} = 10000/480 = 20.8333 A, Z_{base1} = 480/20.8333 = 23.04 Ω.	V_{base2} = 480×10 = 4800 V, S_{base2} = 10 kVA, I_{base2} = 10000/4800 = 2.0833 A, Z_{base2} = 4800/2.0833 = 2304 Ω.	V_{base3} = 4800/20 = 240 V, S_{base3} = 10 kVA, I_{base3} = 10000/240 = 41.6667 A, Z_{base3} = 240/41.6667 = 5.76 Ω.

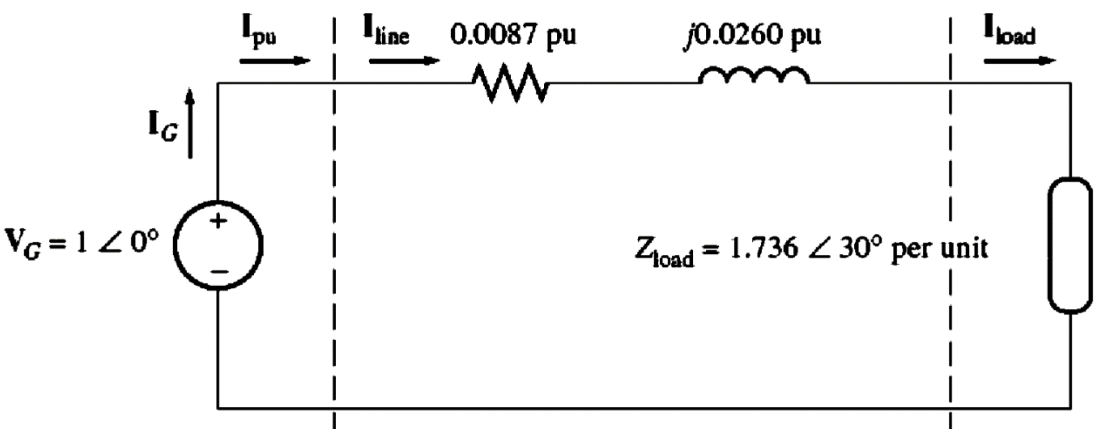
b) To convert a power system to a per-unit system, each component must be divided by its base value in its region of the system.

$V_G = 480\angle 0 \,/\, V_{base1} = 480/480 = 1.0$ p.u

$Z_{line} = (20+j60)/Z_{base2} = 0.0087 + j\,0.0260 = 0.0275\angle 71.5651$

$Z_{load} = 10\angle 30 \,/\, Z_{base3} = 1.7361\angle 30$

Therefore, the equivalent circuit in per unit is given below:



$$I_G = I_{line} = I_{load} = \frac{1}{0.0087 + j0.026 + 1.736\angle 30} = 0.5693\angle -30.5926\ p.u$$

$$V_{load} = 0.5693\angle-30.5926 \times 1.736\angle 30 = 0.9883\angle-0.5926\ p.u$$

$$V_{load} = 0.9883\angle -0.5926 \times 240 = 237.192\ \angle -0.5926\ V$$

The apparent power supplied to the load $S_{load} = V_{load} \times I_{load}^* = 0.4873 + j\ 0.2813$ p.u

The active power = 0.4873 p.u = 4873 W

Reactive power = 0.2813 p.u = 2813 VA

d) power lost in transmission line = $0.5693^2 \times 0.0087 = 2.8197\times10^{-3}$ p.u

power lost in transmission line = $2.8197\times10^{-3} \times 10000 = 28.1969$ W

7. <u>Determination of transformer parameters:</u>

1) Open-Circuit test:

The purpose of this test is to determine the core loss and no-load I_0 which is helpful in finding X_m and R_c. One winding of the transformer –whichever is convenient but usually high voltage winding – is left open and the other is connected to its supply of normal voltage and frequency. A wattmeter *W*, voltmeter *V* and an ammeter *A* are connected in the low voltage winding. With normal voltage applied, normal flux will be set up in the core, hence normal iron losses will occur which are recorded by the wattmeter. As the no-load current I_0 (as measured by ammeter) is small (usually 2 to 5% of rated load current), Cu loss is negligibly small in primary and zero in secondary (it being open). Hence, the wattmeter reading represents practically the core loss under no-load.

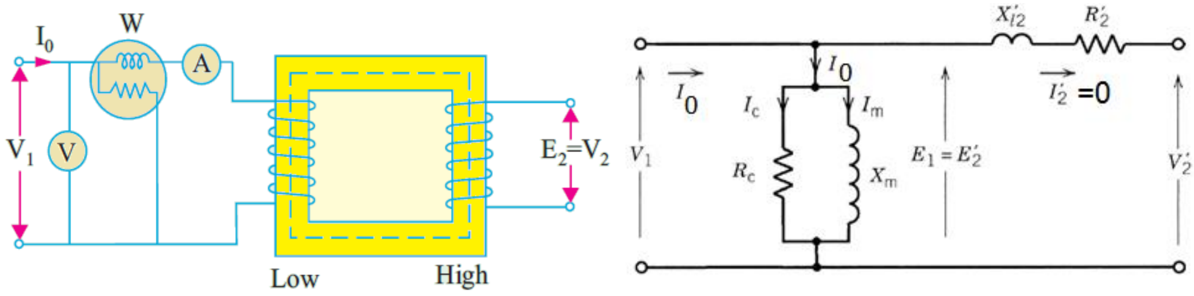


Fig. 23 open-circuit configuration

$$P_{nl} = V_1 I_0 \cos(\phi_0) \quad \rightarrow \quad \cos(\phi_0) = \frac{P_{nl}}{V_1 I_0}$$

$$I_m = I_0 \sin(\phi_o) \quad \rightarrow X_m = \frac{V_1}{I_m}$$

$$I_c = I_0 \cos(\phi_o) \rightarrow R_c = \frac{V_1}{I_c}$$

2) Short-Circuit test:

This test is performed by short-circuiting one winding, usually the LV winding, and applying rated current to the other winding, as shown in Fig. 24. The equivalent circuit of the transformer under short-circuit condition is shown in this figure.

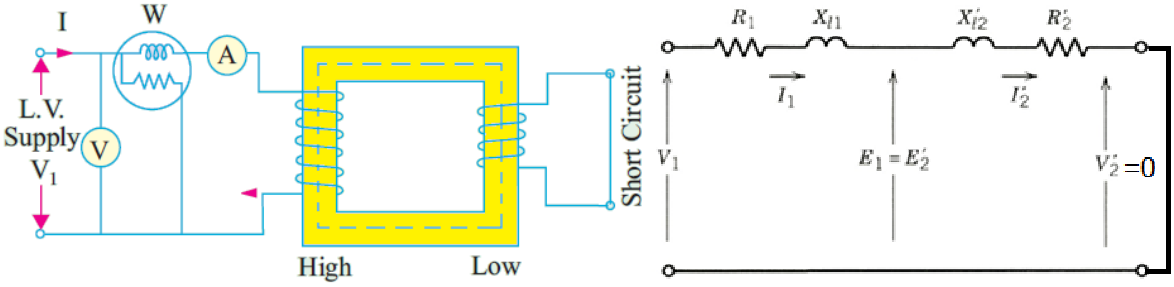


Fig. 24 Short-circuit configuration

Since, in this test, the applied voltage is a small percentage of the normal voltage, the mutual flux Φ produced is also a small percentage of its normal value. Hence, core losses are very small with the result that the wattmeter reading represent the full-load Cu loss or I^2R loss for the whole transformer *i.e.* both primary Cu loss and secondary Cu loss.

$$P_{sc} = V_1 I_1 \cos(\phi_{sc}) \quad \rightarrow \quad \cos(\phi_{sc}) = \frac{P_{sc}}{V_1 I_1} \quad \& \quad Z = \frac{V_1}{I_1}$$

$$R_{eq} = Z \cos(\phi_{sc})$$

$$X_{eq} = Z \sin(\phi_{sc})$$

Example:

Tests are performed on a single-phase, 10 kVA, 2200/220 V, 60 Hz transformer and the following results are obtained:

	Open-Circuit Test (high-voltage side open)	Short-Circuit Test (low-voltage side shorted)
Voltmeter	220 V	150 V
Ammeter	2.5 A	4.55 A
Wattmeter	100 W	215 W

(a) Derive the parameters for the approximate equivalent circuits referred to the low-voltage side and the high-voltage side.

(b) Express the excitation current as a percentage of the rated current.

(c) Determine the power factor for the no-load and short-circuit tests.

$$V_{H(\text{rated})} = 2200 \text{ V}$$
$$V_{L(\text{rated})} = 220 \text{ V}$$
$$I_{H(\text{rated})} = \frac{10{,}000}{2200} = 4.55 \text{ A}$$
$$I_{L(\text{rated})} = \frac{10{,}000}{220} = 45.5 \text{ A}$$
$$V_H I_H|_{\text{rated}} = V_L I_L|_{\text{rated}} = 10 \text{ kVA}$$

$$\text{Power, } P_{oc} = \frac{V_L^2}{R_{cL}}$$

$$R_{cL} = \frac{220^2}{100} = 484\ \Omega$$

$$I_{cL} = \frac{220}{484} = 0.45 \text{ A}$$

$$I_{mL} = (I_L^2 - I_{cL}^2)^{1/2} = (2.5^2 - 0.45^2)^{1/2} = 2.46 \text{ A}$$

$$X_{mL} = \frac{V_L}{I_{mL}} = \frac{220}{2.46} = 89.4\ \Omega$$

The corresponding parameters for the high-voltage side are obtained as follows:

$$\text{Turns ratio } a = \frac{2200}{220} = 10$$

$$R_{cH} = a^2 R_{cL} = 10^2 \times 484 = 48{,}400\ \Omega$$

$$X_{mH} = 10^2 \times 89.4 = 8940\ \Omega$$

$$\text{Power } P_{sc} = I_H^2 R_{eqH}$$

$$R_{eqH} = \frac{215}{4.55^2} = 10.4\ \Omega$$

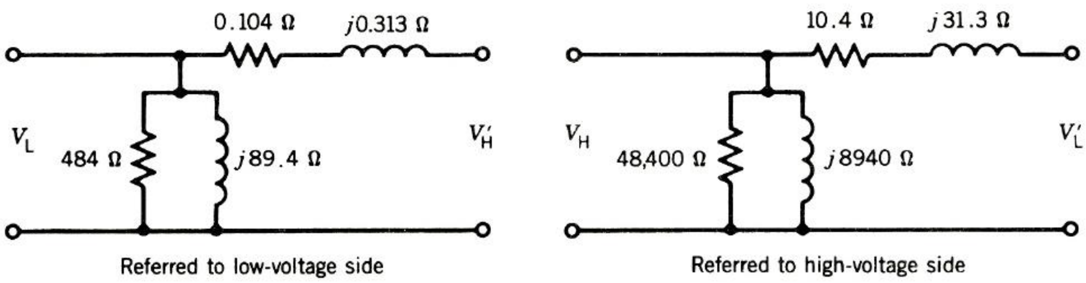
$$Z_{eqH} = \frac{V_H}{I_H} = \frac{150}{4.55} = 32.97\ \Omega$$

$$X_{eqH} = (Z_{eqH}^2 - R_{eqH}^2)^{1/2} = (32.97^2 - 10.4^2)^{1/2} = 31.3\ \Omega$$

The corresponding parameters for the low-voltage side are as follows:

$$R_{eqL} = \frac{R_{eqH}}{a^2} = \frac{10.4}{10^2} = 0.104\ \Omega$$

$$X_{eqL} = \frac{31.3}{10^2} = 0.313\ \Omega$$



(b) From the no-load test the excitation current, with rated voltage applied to the low-voltage winding, is

$$I_\phi = 2.5 \text{ A}$$

This is $(2.5/45.5) \times 100\% = 5.5\%$ of the rated current of the winding.

(c)

$$\text{Power factor at no load} = \frac{\text{power}}{\text{volt-ampere}} = \frac{100}{220 \times 2.5} = 0.182$$

$$\text{Power factor at short-circuit condition} = \frac{215}{150 \times 4.55} = 0.315$$

7. Voltage Regulation:

voltage regulation is used to identify this characteristic of voltage change in a transformer with loading. The voltage regulation is defined as the change in magnitude of the secondary voltage as the load current changes from the no-load to the loaded condition. This is expressed as follows:

$$\text{Voltage regulation} = \frac{|V_2|_{\text{NL}} - |V_2|_{\text{L}}}{|V_2|_{\text{L}}}$$

We can calculate the voltage regulation when the secondary circuit referred to primary

$$\text{Voltage regulation} = \frac{|V'_2|_{\text{NL}} - |V'_2|_{\text{L}}}{|V'_2|_{\text{L}}}$$

$|V'_2|_{\text{L}} = |V'_2|_{\text{rated}}$ and $|V'_2|_{\text{NL}} = |V_1|$

$$\begin{array}{c}\text{Voltage regulation} \\ \text{(in percent)}\end{array} = \frac{|V_1| - |V'_2|_{\text{rated}}}{|V'_2|_{\text{rated}}} \times 100\%$$

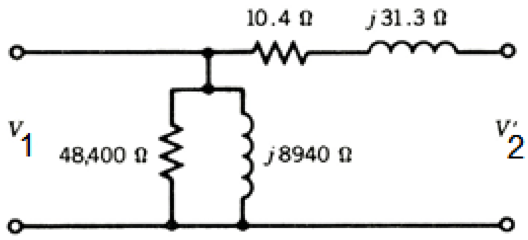
Example:

Consider the transformer in the previous example. Determine the voltage regulation in percent for the following load conditions.

(a) 75% full load, 0.6 power factor lagging.

(b) Draw the phasor diagram for conditions (a).

10 kVA, 2200/220 V transformer and from tests we obtain the equivalent circuit



Referred to high-voltage side

At 75% full laod, then the secondary winding rating is 10000×0.75=7500 VA

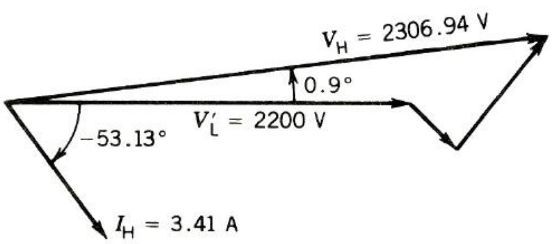
$V_2\ I_2 = 7500 = 220 \times I_2 \rightarrow\ I_2 = 34.0909\ A \rightarrow I_2' = 34.0909 \times 220/2200 = 3.40909\ A$

$V_2' = 220 \times 2200/220 = 2200\ V$

$$\begin{aligned} V_1 &= 2200\angle 0^\circ + 3.41\angle -53.13^\circ(10.4 + j31.3) \\ &= 2200 + 35.46\angle -53.13^\circ + 106.73\angle 90^\circ - 53.13^\circ \\ &= 2200 + 21.28 - j28.37 + 85.38 + j64.04 \\ &= 2306.66 + j35.67 \\ &= 2306.94\angle 0.9^\circ\ V \end{aligned}$$

$$\text{Voltage regulation} = \frac{2306.94 - 2200}{2200} \times 100\% \ = 4.86\%$$

The meaning of 4.86% voltage regulation is that if the load is thrown off, the load terminal voltage will rise from 220 to 230.69 volts.



Lagging PF

8. Efficiency:

Equipment is desired to operate at a high efficiency. Fortunately, losses in transformers are small. Because the transformer is a static device, there are no rotational losses such as windage and friction losses in a rotating machine. The efficiency is defined as follows:

$$\eta = \frac{\text{output power } (P_{\text{out}})}{\text{input power } (P_{\text{in}})} = \frac{P_{\text{out}}}{P_{\text{out}} + \text{losses}}$$

The losses in the transformer are the core loss (P_c) and copper loss (P_{cu}). Therefore,

$$\eta = \frac{P_{\text{out}}}{P_{\text{out}} + P_c + P_{cu}}$$

The copper loss can be determined as:

$$P_{cu} = I_1^2 R_1 + I_2^2 R_2$$

$$P_{\text{out}} = V_2 I_2 \cos\theta_2$$

$$\eta = \frac{V_2 I_2 \cos\theta_2}{V_2 I_2 \cos\theta_2 + P_c + I_2^2 R_{eq2}}$$

Therefore, efficiency depends on load current (I_2) and load power factor ($\cos\theta_2$).

8.1 Maximum Efficiency

$$\text{Primary input} = V_1 I_1 \cos\theta_1$$

$$\eta = \frac{V_1 I_1 \cos\theta_1 - \text{losses}}{V_1 I_1 \cos\theta_1} = \frac{V_1 I_1 \cos\theta_1 - I_1^2 R_{01} - W_i}{V_1 I_1 \cos\theta_1}$$

$$= 1 - \frac{I_1 R_{01}}{V_1 \cos\theta_1} - \frac{W_i}{V_1 I_1 \cos\theta_1}$$

W_i is the iron loss and $I_1^2 R_{01}$ is the cupper loss referred to primary

For constant values of the terminal voltage V_2 and load power factor angle θ_2, the maximum efficiency occurs when

$$\frac{d\eta}{dI_1}$$

Differentiating both sides with respect to I_1, we get

$$\frac{d\eta}{dI_1} = 0 - \frac{R_{01}}{V_1 \cos\theta_1} + \frac{W_i}{V_1 I_1^2 \cos\theta_1}$$

For η to be maximum, $\frac{d\eta}{dI_1} = 0$. Hence, the above equation becomes

$$\textbf{Cu loss} = \textbf{Iron loss}$$

The output current corresponding to maximum efficiency is

$$I_2 = \sqrt{(W_i/R_{02})}$$

The efficiency at any load is given by

$$\eta = \frac{x \times \text{full-load kVA} \times \text{p.f.}}{(x \times \text{full-load kVA} \times \text{p.f.}) + x^2 \times W_{cu} + W_i} \times 100$$

where x = ratio of actual to full-load kVA

W_i = iron loss in kW ; W_{cu} = Cu loss in kW.

<u>Example:</u>

In a 25-kVA, 2000/200 V, single-phase transformer, the iron and full-load copper losses are 350 and 400 W respectively. Calculate the efficiency at unity power factor on

(i) full load **(ii)** half full-load.

(iii) at which loading factor does max efficiency occur? And what is its value?

At Full-load (x=1) Unity p.f.

$$\eta = \frac{1 \times 25 \times 1}{1 \times 25 \times 1 + 400 + 350} \times 100 = 97.087\%$$

At half full-load (x=0.5) Unity p.f.

$$\eta = \frac{0.5 \times 25 \times 1}{0.5 \times 25 \times 1 + 0.5^2 \times 400 + 350} \times 100 = 96.525\%$$

For maximum efficiency, P_{cu} must be 350 (equal the iron loss),

$$350 = x^2(400) \quad \rightarrow x = 0.9354$$

$$\eta_{max} = \frac{0.9354 \times 25 \times 1}{0.9354 \times 25 \times 1 + 350 + 350} \times 100 = 97.0937\%$$

<u>Example:</u>

A 5-kVA, 2,300/230-V, 50-Hz transformer was tested for the iron losses with normal excitation and Cu losses at full-load and these were found to be 40 W and 112 W respectively. Calculate the efficiencies of the transformer at 0.8 power factor for the following kVA outputs:

1.25 2.5 3.75 5.0 6.25 7.5

Then obtain the maximum efficiency.

#	P_{out}(KVA)	x	x^2 P_{cuFL}	P_{iron}	η
1	1.25	0.25	7	40	$\frac{0.25 \times 5000 \times 0.8}{1250 \times 0.8 + 7 + 40} = 95.511$
2	2.5	0.5	28	40	$\frac{0.5 \times 5000 \times 0.8}{2500 \times 0.8 + 28 + 40} = 96.712$

3	3.75	0.75	63	40	$\frac{0.75 \times 5000 \times 0.8}{3750 \times 0.8 + 63 + 40} = 96.681$
4	5	1	112	40	$\frac{1 \times 5000 \times 0.8}{5000 \times 0.8 + 112 + 40} = 96.3391$
5	6.25	1.25	175	40	$\frac{1.25 \times 5000 \times 0.8}{6250 \times 0.8 + 175 + 40} = 95.8773$
6	7.5	1.5	252	40	$\frac{1.5 \times 5000 \times 0.8}{7500 \times 0.8 + 252 + 40} = 95.3592$

For maximum efficiency, P_{cu} must be 40 (equal the iron loss),

$$40 = x^2(112) \rightarrow x = 0.597614$$

$$\eta_{max} = \frac{0.597614 \times 5000 \times 0.8}{0.597614 \times 5000 \times 0.8 + 40 + 40} \times 100 = 96.7617\%$$

Example:

A 200-kVA transformer has an efficiency of 98% at full load. If the max. efficiency occurs at three quarters of full-load, calculate the efficiency at half load. Assume p.f. 0.8 at all loads.

As given, the transformer has a F.L. efficiency of 98 % at 0.8 p.f.

F.L. output = 200 × 0.8 = 160 kW ; F.L. input = 160/0.98 = 163.265 kW

F.L. losses = 163.265 – 160 = 3.265 kW

This loss consists of F.L. Cu loss x and iron loss y.

∴ $x + y = 3.265$ kW ...(*i*)

It is also given that η_{max} occurs at three quarters of full-load when Cu loss becomes equal to iron loss.

∴ Cu loss at 75 % of F.L. = $x\,(3/4)^2 = 9\,x/16$

Since y remains constant, hence $9\,x/16 = y$ ...(*ii*)

Substituting the value of y in Eqn. (*i*), we get $x + 9\,x/16 = 3265$ or $x = 2090$ W; $y = 1175$ W

Half-load Unity p.f.

Cu loss = $2090 \times (1/2)^2 = 522$ W ; total loss = 522 + 1175 = 1697 W

Output = 100 × 0.8 = 80 kW ; η = 80/81.697 = 0.979 or **97.9 %**

8.2 All-Day Efficiency

The transformer connected to the utility that supplies power to your house and the locality is called a distribution transformer. Such transformers are connected to the power system 24 hours a day and operate well below the rated power output for most of the time. The efficiency performance of a distribution transformer is the "all-day" or "energy" efficiency of the transformer. This is defined as follows:

$$\eta_{AD} = \frac{\text{energy output over 24 hours}}{\text{energy input over 24 hours}}$$

$$= \frac{\text{energy output over 24 hours}}{\text{energy output over 24 hours} + \text{losses over 24 hours}}$$

Example:

Find the all-day efficiency of a 50kVA distribution transformer having full load efficiency of 94 % and full-load copper losses are equal to the constant iron losses. The loading of the transformer is as follows; the power factor is unity.

(i) No load for 10 hours (ii) Half load for 5 hours

(iii) 25 % load for 6 hours (iv) Full load for 3 hours.

At full load unity p.f.

$$\text{efficiency} = 94\,\% = \frac{50,000}{50,000 + 2P_i}$$

$$2P_i = \left[\frac{50,000}{0.94} - 50,000\right], \quad \text{or} \quad P_i = \frac{1}{2} \times 50,000\left[\frac{1-0.94}{0.94}\right],$$

$$P_i = 25,000 \times \frac{0.06}{0.94} = 1596 \text{ Watts}$$

Hence, full load Cu-losses = 1596 Watts

And the iron loss (fixed loss) = 1596 Watts

#	x	Time	E_{cu} (Wh)	E_{output} (Wh)	E_{iron} (Wh)
1	0	10	0	0	1596×10
2	0.5	5	$(0.5)^2 \times 1596 \times 5 = 1995$	0.5×50000×5=125000	1596×5
3	0.25	6	$(0.25)^2 \times 1596 \times 6 = 598.5$	0.25×50000×6=75000	1596×6
4	1	3	$1596 \times 3 = 4788$	50000×3=150000	1596×3
Total			7381.5	350000	38304

Then, all day efficiency is calculated as:

$$\frac{350000}{350000 + 7381.5 + 38304} = 88.4541\%$$

Example:

A 50 kVA, 2400/240 V transformer has a core loss $P_c = 200$ W at rated voltage and a copper loss $P_{cu} = 500$ W at full load. It has the following load cycle:

% Load	**0.0%**	**50%**	**75%**	**100%**	**110%**
Power factor		1	0.8 lag	0.9 lag	1
Hours	6	6	6	3	3

Determine the all-day efficiency of the transformer.

#	x	Time	E_{cu} (Wh)	E_{output} (kWh)	E_{iron} (Wh)
1	0	6	0	0	200×6
2	0.5	6	$(0.5)^2 \times 500 \times 6 = 750$	0.5×50000×6=150	200×6
3	0.75	6	$(0.75)^2 \times 500 \times 6 = 1687.5$	0.75×50000×0.8×6=180	200×6
4	1	3	$500 \times 3 = 1500$	50000×0.9×3=135	200×3
5	1.1	3	$(1.1)^2 \times 500 \times 3 = 1815$	1.1×50000×3=165	200×3
Total			5752.5	630	4800

Then, all day efficiency is calculated as:

$$\frac{630000}{630000 + 5752.5 + 4800} = 98.3526\%$$

Example:

A 10 kVA, 1-ph transformer has a core-loss of 400 W and full load ohmic loss of 100 W. The daily variation of load on the transformer is as follows:

6 a.m. to 1 p.m. 3 kW at 0.60 p.f.

1 p.m. to 5 p.m. 8kW at 0.8 p.f.

5 p.m. to 1 a.m. full load at u.p.f.

1 a.m. to 6 a.m. no load

Determine all day efficiency of the transformer

S.N.	*Number of hours*	$x = \frac{\text{load kVA}}{\text{Xmer Rating}}$	$x^2 P_c$ *in kW*	*Output in kWh*	*Copper Loss in kWh*
1	7	$\frac{3/0.6}{10} = 0.5$	$0.50^2 \times 0.10 = 0.025$	$3 \times 7 = 21$	$0.025 \times 7 = 0.175$
2	4	$\frac{8/0.8}{10} = 1.0$	0.10	$8 \times 4 = 32$	$0.1 \times 4 = 0.40$
3	8	$\frac{10/1}{10} = 1.0$	0.10	$10 \times 8 = 80$	$0.1 \times 8 = 0.8$
4	5	Zero	Zero	Zero	Zero
			Output in kWh	$21 + 32 + 80 = 133$	
			Ohmic Loss, in kWh		$0.175 + 0.40 + 0.80 = 1.375$
		Core loss during 24 Hrs $= \frac{400}{1000} \times 24 = 0.96$ kWh			

$$\text{Hence, Energy efficiency } (= \text{All day Efficiency}) = \frac{133}{133 + 1.375 + 0.96} \times 100 = 98.3\,\%$$

Example:

When a 94-kVA single-phase transformer was tested using open-circuit and short-circuit tests, the following data were obtained: On open circuit, the power consumed was 940 W and on short-circuit the power consumed was 1350 W.

a) Calculate the efficiency of the transformer at **full-load** & **half full-load** when working at unity p.f.,

b) Calculate the maximum efficiency at a unity p.f.,

c) The transformer undergoes a daily load cycle as given in table below, determine the all-day efficiency.

Time Duration (minutes)	300	180	240	300	240	180
Loading (%)	25%	75%	100%	115%	50%	0%
Power Factor	Unity	0.8 lag	0.77 lag	0.9 lag	0.68 lag	---

From O.C. test → P_{iron} = 940 W,

From S.C. test → $P_{cu\ F.L}$ = 1350 W,

$P_{out\ F.L}$ = 94000 VA

At Full-load (x=1) Unity p.f.

$$\eta = \frac{94000}{94000 + 940 + 1350} \times 100 = 97.6218\%$$

At Half Full-load (x=0.5) Unity p.f.

$$\eta = \frac{94000 \times 0.5}{94000 \times 0.5 + 940 + (0.5)^2 \times 1350} \times 100 = 97.3538\%$$

For maximum efficiency, P_{cu} must be 940 (equal the iron loss), at unity p.f.

$$940 = x^2(1350) \ \rightarrow x = 0.8344$$

$$\eta_{max} = \frac{0.8344 \times 94000}{0.8344 \times 94000 + 940 + 940} \times 100 = 97.6593\%$$

#	X	p.f	Time	E_{out} (kWh) $x \times P_{out\ FL} \times$p.f.$\times$Time	E_{cu} (Wh) $x^2 \times P_{cu\ FL} \times$Time	E_{iron} (Wh) $P_{iron} \times$Time
1	0.25	1	5	0.25×94×1×5=117.5	0.25^2×1350×5=421.875	940×5
2	0.75	0.8	3	0.75×94×0.8×3=169.2	0.75^2×1350×3=2278.125	940×3
3	1.0	0.77	4	1×94×0.77×4=289.52	1×1350×4=5400	940×4
4	1.15	0.9	5	1.15×94×0.9×5=486.45	1.15^2×1350×5=8926.875	940×5

5	0.5	0.68	4	0.5×94×0.68×4=127.84	0.5^2×1350×4=1350	940×4
6	0	--	3	0	0	940×3
Total				1190.51	18376.875	22560

Then, all day efficiency is calculated as:

$$\frac{1190510}{1190510 + 18376.875 + 22560} = 96.6757\%$$

Example:

A 100-kVA distribution transformer has a maximum efficiency of 98 % at 50 % full-load and unity power factor. Determine its iron losses and full-load copper losses.

Determine its all-day efficiency if the transformer undergoes a daily load cycle as follows:

Load	*Power factor*	*Load duration*
100 kVA	1.0	8 hrs
50 kVA	0.8	6 hrs
No load		10 hrs

Since the max efficiency occurs at half full load, then $x = 0.5$

$$P_{iron} = x^2 P_{cu\,FL} = 0.25 P_{cu\,FL}$$

Also, the maximum efficiency at 0.5 FL = 0.98, then

$$0.98 = \frac{0.5 \times 100000}{0.5 \times 100000 + P_{cu\,FL} + 0.25 P_{cu\,FL}}$$

$$50000 = 49000 + 0.49\,P_{cu\,FL}$$

$$P_{cu\,FL} = 2040.8163\ \text{W} \quad \#\#\#\#\#$$

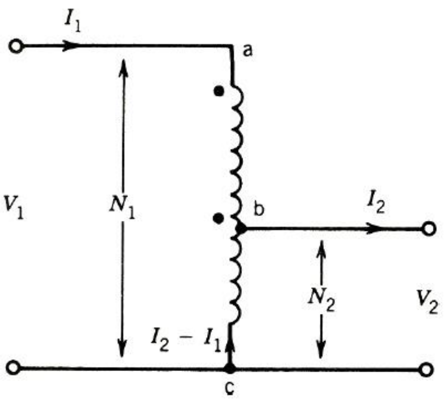
$$P_{iron} = 0.25 \times 2040.8163 = 510.2041\ \text{W}\ \#\#\#\#$$

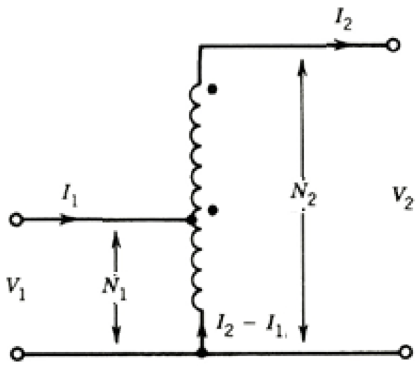
#	*x*	Time	E_{cu} (Wh)	E_{output} (kWh)	E_{iron} (Wh)
1	1	8	$2040.8163 \times 8 = 16326.5304$	100×1×8=800	510.2041×8
2	0.5	6	$(0.5)^2 \times 2040.8163 \times 6 = 3061.2245$	0.5×100×0.8×6=240	510.2041×6
4	0	10	0	0	510.2041×10
Total			19387.7549	1040	12244.8984

$$\eta_{all\ day} = \frac{1040000}{1040000 + 19387.7549 + 12244.8984} = 97.0482\%$$

AutoTransformers

On some cases it is desirable to change voltage levels by only a small amount. For example, from 110 to 120 V or from 13.2 to 13.8 kV. In such circumstances, it is use autotransformers. In such kind of transformers, a common winding as shown in Fig. 23 is mounted on a core and the secondary is taken from a tap on the winding. In contrast to the two-winding transformer, the primary and secondary of an autotransformer are physically connected.





Equivalent circuit: (Step-Down) (Step-Up)

$$\frac{V_2}{V_1} = \frac{N_2}{N_1} = a, \qquad \frac{I_2}{I_1} = \frac{1}{a}$$

Fig. 23 Autotransformer

However, the basic principle of operation is the same as that of the two-winding transformer. Since all the turns link the same flux in the transformer core,

Example

A transformer has a primary voltage rating of 11500 volts and secondary voltage rating of 2300 volts. Two windings are connected in series and the primary is connected to a

supply of 11500 volts, to act as a step-up auto transformer. Determine the voltage output of the transformer. If the two-winding transformer is rated at 115 kVA, what will be the kVA rating of the auto-transformer?

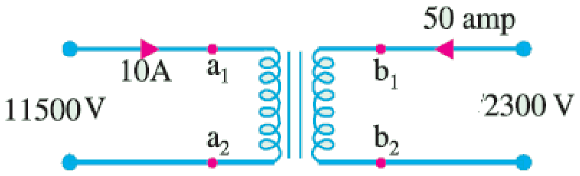
11500/2300 V, 115 kVA, transformer has the current ratings of 10 A and 50 A.

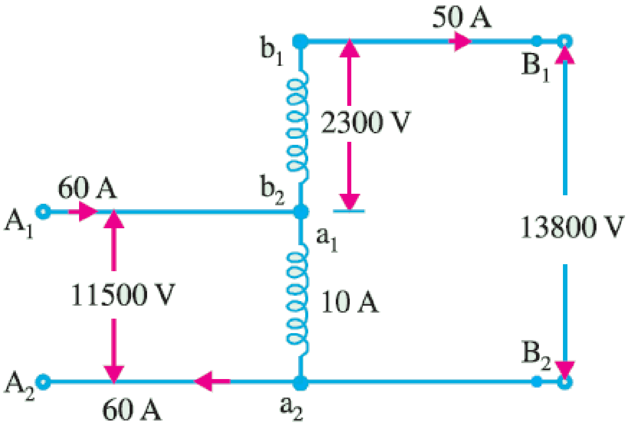
the step-up connection is shown. Winding currents have to be at the same rated values. The voltage obtainable at $B1 - B2$ is 13800 V, and from $b1$ a load-current of 50 A can be supplied.

Total power delivered to the load = 13800 × 50 = 690000 VA.

Total power transferred magnetically = 11500×10 = 2300×50 = 115000 VA

Total power transferred Electrically = 690000 – 115000 = 575000 VA



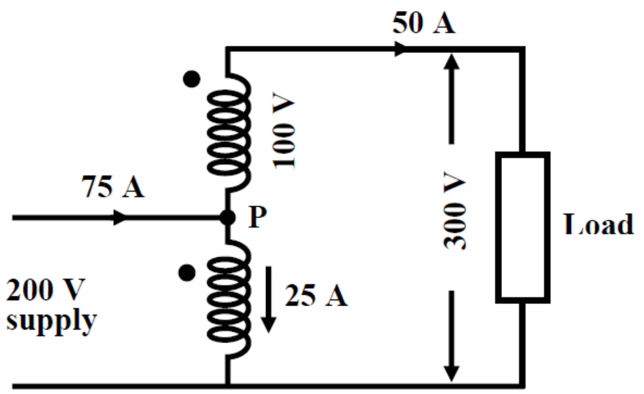


Example

A 5kVA, 200 V/ 100 V, 50 Hz, single phase ideal two-winding transformer is used to step up a voltage of 200 V to 300 V by connecting it like an auto transformer. Show the

connection diagram to achieve this. Calculate the maximum kVA that can be handled by the autotransformer (without over loading any of the HV and LV coil). How much of this kVA is transferred magnetically and how much is transferred by electrical conduction?

To connect a two-winding transformer as an autotransformer, it is essential to know the dot markings on the two coils. The coils are to be now series connected appropriately so as to identify clearly between which two terminals to give supply and between which two to connect the load. Since the input voltage here is 200 V, supply must be connected across the HV terminals. The induced voltage in the LV side in turn gets fixed to 100 V. But we require 300 V as output, so LV coil is to be connected in additive series with the HV coil. This is what has been shown in figure below.



$$\text{Rated voltage of HV coil is} = 200 \text{ V}$$

$$\text{Rated voltage of LV coil is} = 100 \text{ V}$$

$$\text{Phase turns ratio is } a = 200/100 = 2$$

$$\text{Rated current of each HV coil is} = 5000/200 = 25 \text{ A}$$

$$\text{Rated current of each LV coil is} = 5000/100 = 50 \text{ A}$$

$$\text{Current drawn form the supply} = 15000/200 = 75 \text{ A}$$

$$\text{Current through HV coil } I_{HV} = 75 - 50 = 25 \text{ A}$$

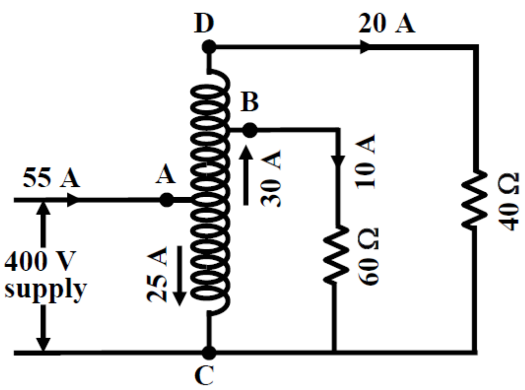
$$\text{Output kVA} = 300 \times 50 \text{ VA} = 15 \text{ kVA}$$

$$\text{input kVA} = 200 \times 75 \text{ VA} = 15 \text{ kVA}$$

$$\text{kVA transferred magnetically} = \text{kVA of either HV or LV coil}$$
$$= 200 \times 25 \text{ VA} = 100 \times 50 \text{ VA} = 5 \text{ kVA}$$
$$\therefore \text{ kVA transferred magnetically} = 5kVA$$
$$\text{kVA transferred electrically} = \text{total kVA transferred} - \text{kVA transferred magnetically}$$
$$= 15 - 5 = 10 \text{ kVA}$$

Example:

An autotransformer has a coil with total number of turns $N_{CD} = 200$ between terminals C and D. It has got one tapping at A such that $N_{AC} = 100$ and another tapping at B such that $N_{BA} = 50$. Calculate currents in various parts of the circuit and show their directions when 400 V supply is connected across AC and two resistive loads of 60Ω & 40Ω are connected across BC and DC respectively.



$$\text{Supply voltage across AC, } V_{AC} = 400\text{V}$$
$$\text{Number of turns between A \& C } N_{AC} = 100$$
$$\text{Voltage per turn} = 400/100 = 4\text{V}$$
$$\text{Voltage across the } 40\Omega \text{ load} = N_{DC} \times \text{Voltage per turn}$$
$$= 200 \times 4 = 800\text{V}$$
$$\text{So, current through } 40\Omega = 800/40 = 20\text{A}$$
$$\text{Voltage across the } 60\Omega \text{ } load = N_{BC} \times \text{Voltage per turn}$$
$$= 150 \times 4 = 600 \text{ V}$$
$$\text{So, current through } 60\Omega = 600/60 = 10\text{A}$$

Total output kVA will be the simple addition of the kVAs supplied to the loads i,e.,
$(600 \times 10 + 800 \times 20)$ VA = 22000 VA = 22 kVA

Assuming the autotransformer to be ideal, input kVA must also be 22 kVA.

The current drawn from the supply.

Current drawn from the supply = 22000/400 = 55 A

Current in DB part of the winding I_{BD} = 20 A

Applying KCL at B, current in AB part I_{AB} = 20 + 10 = 30 A

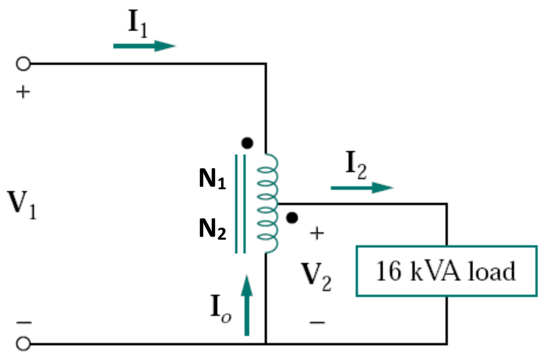
Applying KCL at A, current in AC part I_{AC} = 55 – 30 = 25 A

Example:

For autotransformer shown in figure below, Assuming V_1 = 1250 v and V_2 = 800 V,

i) Find the number of turns N_1 and N_2,

ii) Calculate the currents I_1, I_2 and I_o.



$$\frac{V_1}{V_2} = \frac{N_1 + N_2}{N_2} = \frac{1250}{800}$$

N_2 = 800 and $N_1 + N_2$ = 1250, then N_1 = 1250 – 800 = 450

At load side $V_2 I_2 = 16\times10^3$

I_2 = 20 A

$$\frac{I_2}{I_1} = \frac{N_1 + N_2}{N_2} = \frac{1250}{800}$$

I_1 = 20 × 800/1250 = 12.8 A

$I_o = I_2 - I_1$ = 20 – 12.8 = 7.2 A

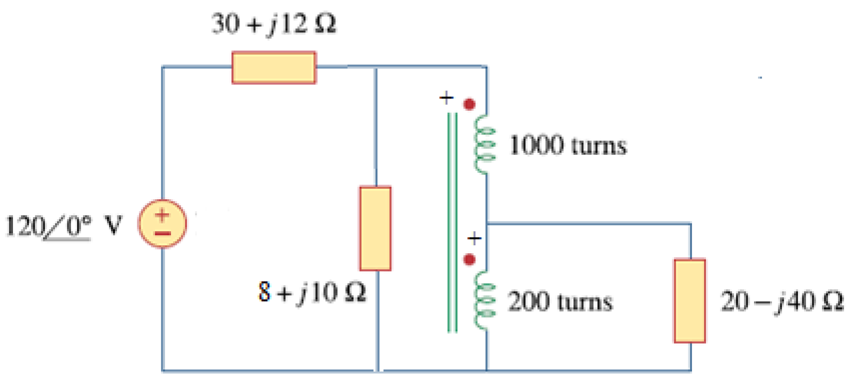
Example:

For the ideal autotransformer shown in figure below, Determine:

a) the primary voltage and current,

b) the secondary voltage and current,

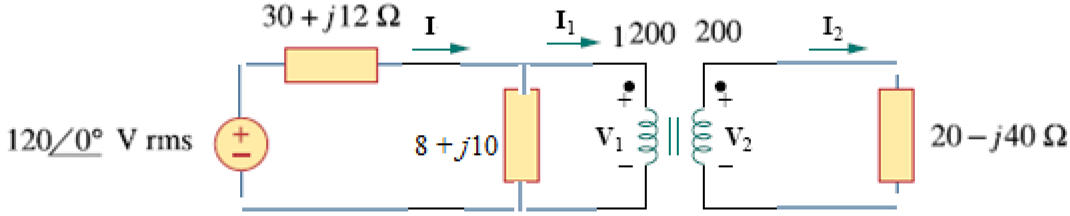
c) the complex power delivered by the source.



This is a step down auto transformer whose turns ratio is given by

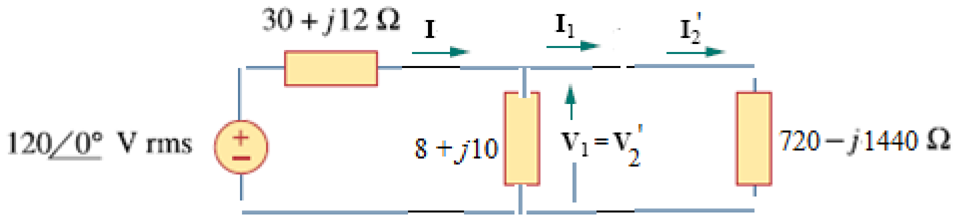
$$\frac{V_1}{V_2} = \frac{N_1}{N_2} = \frac{1000+200}{200} = \frac{1200}{200} = 6$$

The auto transformer can be converted to conventional two winding transformer



Z_L transferred to Primary Z_L' = (20-j40)× $\left(\frac{1200}{200}\right)^2 = 720 - j1440\ \Omega$

After referring the secondary circuit to the primary circuit, the equivalent circuit becomes



$$Z_{eq} = (720 - j1440)//(8 + j10)\ + 30 + j12 = 43.982\angle 29.98°\ \Omega$$

$$I = \frac{120}{43.982\angle 29.98} = \ 2.7284\angle - 29.98\ A$$

Using current divider to obtain the current I_2' which equal I_1 as:

$$I_2' = I_1 = 2.7284 \angle -29.98 \; \frac{8 + j10}{728 - j1430} = 0.0218\angle 84.38 \; A$$

The primary voltage V_1 can be obtained as:

$$V_1 = V_2' = I_2' Z_L' = \; 0.0218\angle 84.38 \; (720 - j1440) = 35.057\angle 20.95 \; V$$

$$\frac{V_1}{V_2} = \frac{I_2}{I_1} = \frac{1200}{200} = 6$$

From above equation,

$$V_2 = \frac{V_1}{6} = \frac{35.057\angle 20.95}{6} = 5.843\angle 20.95 \;\; V$$

$$I_2 = I_1 \times 6 = \; 0.0218\angle 84.38 \times 6 = 0.1308\angle 84.38 \;\; A$$

Therefore, the primary and secondary Voltage & Current are given below:

Primary		Secondary	
Voltage V_1	Current I_1	Voltage V_2	Current I_2
$35.057\angle 20.95$	$0.0218\angle 84.38$	$5.843\angle 20.95 \;\; V$	$0.1308\angle 84.38 \;\; A$

The complex power at the source side = $V_1 \, I^* = 120\angle 0 \;\; \times 2.7284 \angle 29.98$

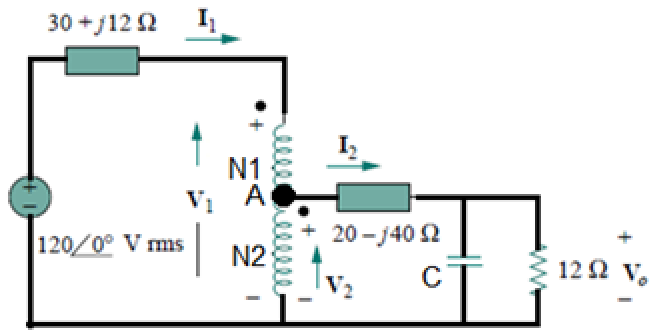
$= 327.057 \angle 29.98 \; VA$

<u>Example:</u>

The autotransformer circuit, shown in figure below, is working at 60 Hz. The shunt capacitor C=294.73 μF & the slider is fixed at point A such that N_1 =250 turns and N_2 =150 turns.

a) Calculate I_1 & I_2 and V_o.

b) If the slider is moved down by 50 turns, calculate the new values of I_1 & I_2 and V_o.

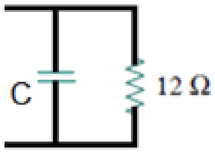


C = 294.73 μF

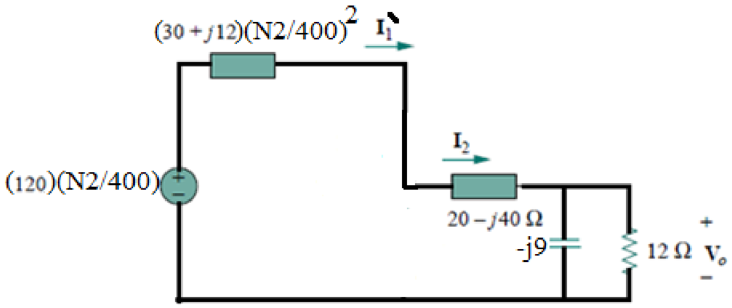
At *f* = 60 Hz

$$X_c = -j\frac{1}{2\pi \times 60 \times 294.73 \times 10^{-6}} = -j9$$

-j9 // 12 = 4.32 – j5.76 Ω



Referring the primary circuit to secondary circuit



For the slider position at N2 = 150

The total impedance Z = 4.21875+j1.6875+20-j40+4.32-j5.76 = 28.53875-j44.0725 Ω

$$I_1{}' = I_2 = \frac{45}{28.53875 - j44.0725} = 0.857\angle 57.075\ A$$

As we know

$$I_1' = I_1\left(\frac{400}{N_2}\right) \qquad I_1 = I_1{}'\left(\frac{N_2}{400}\right)$$

$$I_1 = 0.857\angle 57.075\left(\frac{150}{400}\right) = 0.321375\angle 57.075$$

$$V_o = I_2\,(4.32 - j5.76) = 6.17\angle 3.945\ V$$

For the slider position is moved down by 50 turns, therefore N2 = 100

The total impedance Z = 1.875+j0.75+20-j40+4.32-j5.76 = 26.195 – j45.01 Ω

$$I_1{}' = I_2 = \frac{30}{26.195 - j45.01} = 0.576\angle 59.8\ A$$

As we know

$$I_1' = I_1 \left(\frac{400}{N_2}\right) \qquad\qquad I_1 = I_1{}' \left(\frac{N_2}{400}\right)$$

$$I_1 = 0.576\angle 59.8 \left(\frac{100}{400}\right) = 0.144\angle 59.8\ A$$

$$V_o = I_2\,(4.32 - j5.76) = 4.147\angle 6.67\ V$$

Three-Phase Transformers:

Changing the voltage of a 3-phase system can be done with a 3-phase transformer or with single-phase transformers. The 3-phase transformer has one core with three sets of windings. A primary and a secondary winding are placed one on top of the other on each of the three legs of the core as shown in Fig. 24.

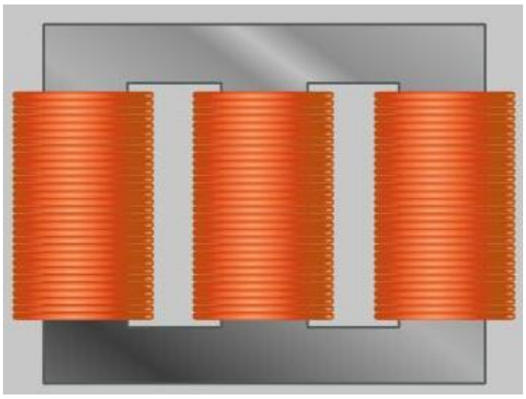
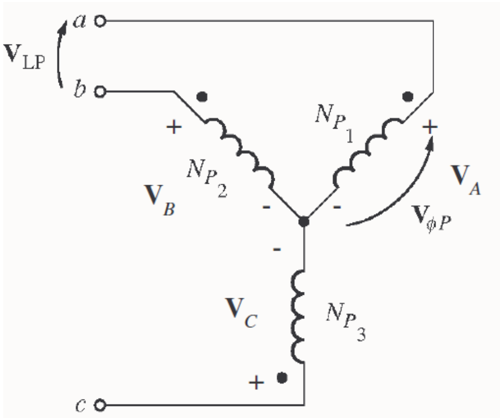
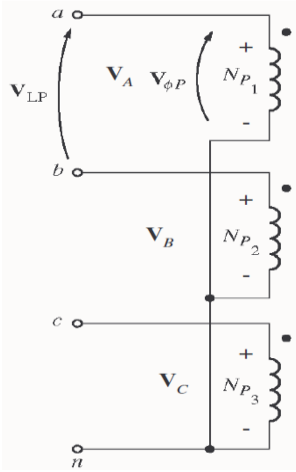


Fig. 24 3-phase transformer: Core type Shell type

If the primary windings are connected in wye (Y) configurations:



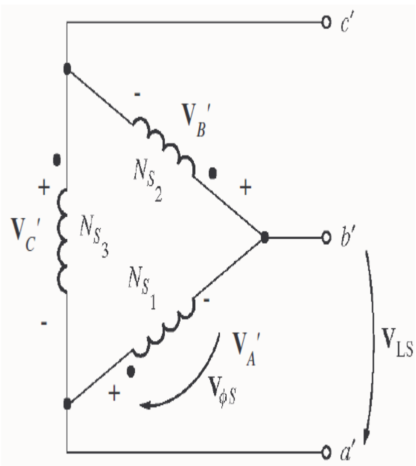


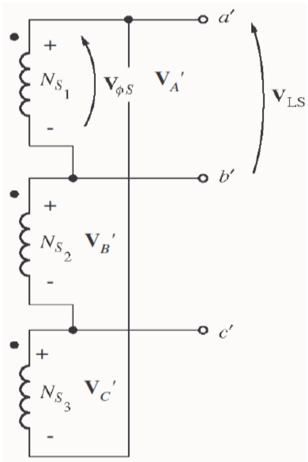
$$V_{\phi P} = \frac{V_L}{\sqrt{3}}$$

$$I_{\phi P} = I_L$$

$$S_{\phi P} = \frac{S}{3}$$

If the secondary windings are connected in delta (Δ) configurations:





$$V_{\phi P} = V_L$$

$$I_{\phi P} = \frac{I_L}{\sqrt{3}}$$

$$S_{\phi P} = \frac{S}{3}$$

The common connections of the 3-phase transformers are:

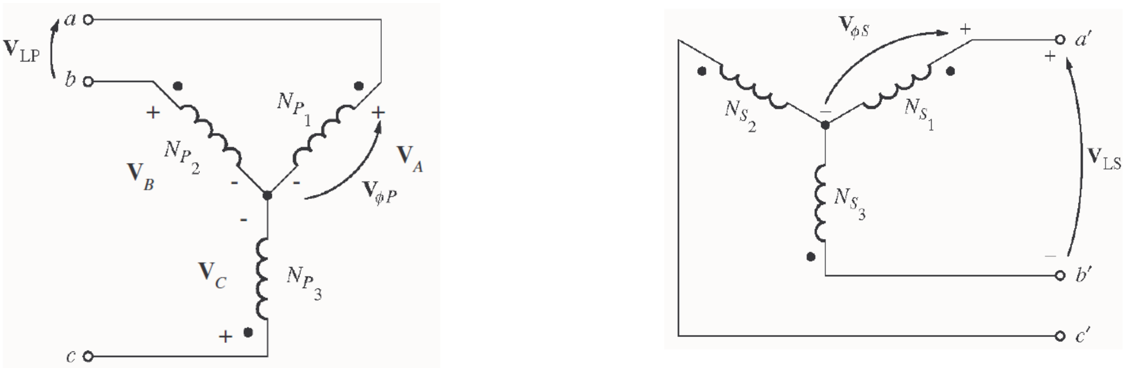
Y-Δ: This connection is commonly used to step down a high voltage to a lower voltage.

Δ-Y: This connection is commonly used to step up voltage.

Δ-Δ: This connection has the advantage that one transformer can be removed for repair
and the remaining two can continue to deliver three-phase power at a reduced rating of 58% of that of the original bank. This is known as the open-delta or V connection.

Y-Y: This connection is rarely used because of problems with the exciting current and induced voltages.

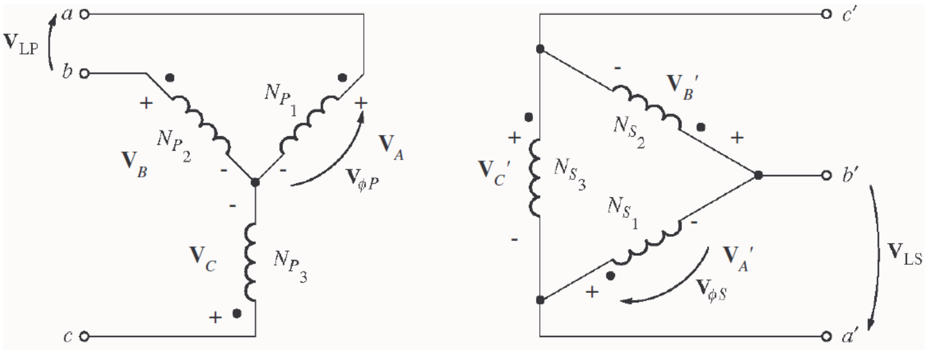
In case of Y-Y connection:



The voltage ratio of the transformer is:

$$\frac{Secondary\ line-to-line\ voltage}{Primary\ line-to-line\ voltage}=\frac{V_{LS}}{V_{LP}}=\frac{\sqrt{3}V_{\emptyset S}}{\sqrt{3}V_{\emptyset P}}=a$$

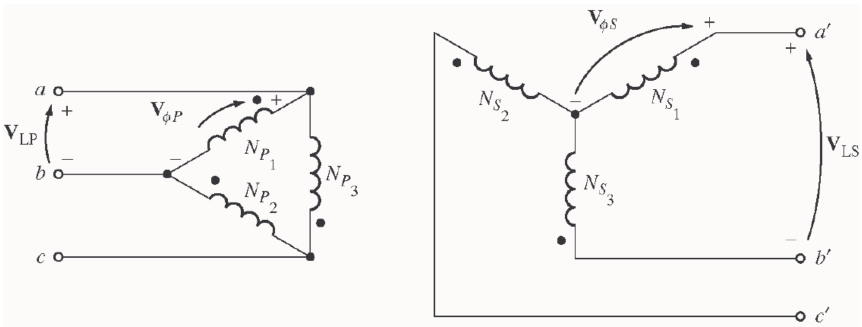
In case of Y-Δ connection:



The voltage ratio of the transformer is:

$$\frac{Secondary\ line-to-line\ voltage}{Primary\ line-to-line\ voltage}=\frac{V_{LS}}{V_{LP}}=\frac{V_{\emptyset S}}{\sqrt{3}V_{\emptyset P}}=\frac{a}{\sqrt{3}}$$

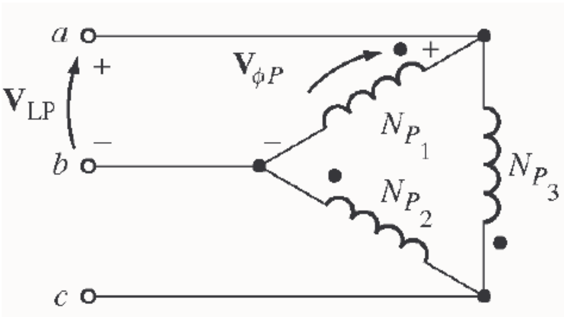
In case of Δ-Y connection:

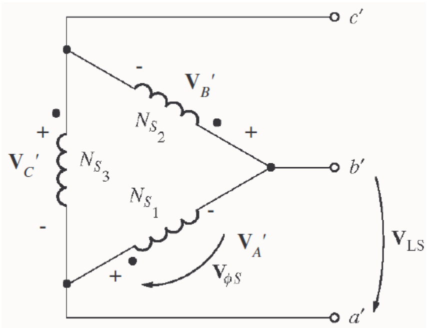


The voltage ratio of the transformer is:

$$\frac{Secondary\ line-to-line\ voltage}{Primary\ line-to-line\ voltage}=\frac{V_{LS}}{V_{LP}}=\frac{\sqrt{3}V_{\emptyset S}}{V_{\emptyset P}}=\sqrt{3}\,a$$

In case of Δ-Δ connection:





The voltage ratio of the transformer is:

$$\frac{Secondary\ line-to-line\ voltage}{Primary\ line-to-line\ voltage}=\frac{V_{LS}}{V_{LP}}=\frac{V_{\emptyset S}}{V_{\emptyset P}}=a$$

Example:

Three single-phase, 50 kVA, 2300/230 V, 60 Hz transformers are connected to form a three-phase, 4000/230V transformer bank. The equivalent impedance of each transformer referred to low voltage is 0.012+j0.016 Ω. The 3-phase transformer supplies a 3-phase, 120 kVA, 230 V, 0.85 PF (lag) load.

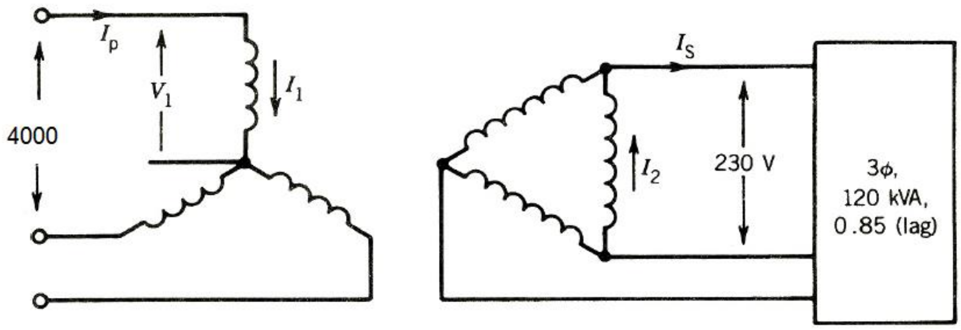
(a) Draw a schematic diagram showing the transformer connection.

(b) Determine the transformer winding currents.

(c) Determine the primary voltage (line-to-line) required.

(d) Determine the voltage regulation.

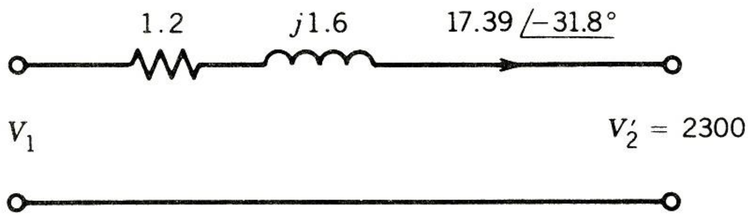
The connection diagram is shown below. The HV windings are to be connected in wye so that the primary can be connected to the 4000 V supply. The LV winding is connected in delta to form a 230 V system for the load.



$$I_s = \frac{120000}{\sqrt{3}\,230} = 301.2262\ A$$

$$I_2 = \frac{301.2262}{\sqrt{3}} = 173.913\ A$$

$$\frac{I_1}{I_2} = \frac{I_1}{173.913} = \frac{230}{2300} \rightarrow I_1 = 17.3913\ A$$



(c) Computation can be carried out on a per-phase basis.

$$Z_{eq1} = (0.012 + j0.016)10^2 = 1.2 + j1.6\ \Omega$$

$$\phi = \cos^{-1} 0.85 = 31.8°$$

$$V_1 = 2300\underline{/0°} + 17.39\underline{/-31.8°}(1.2 + j1.6)$$

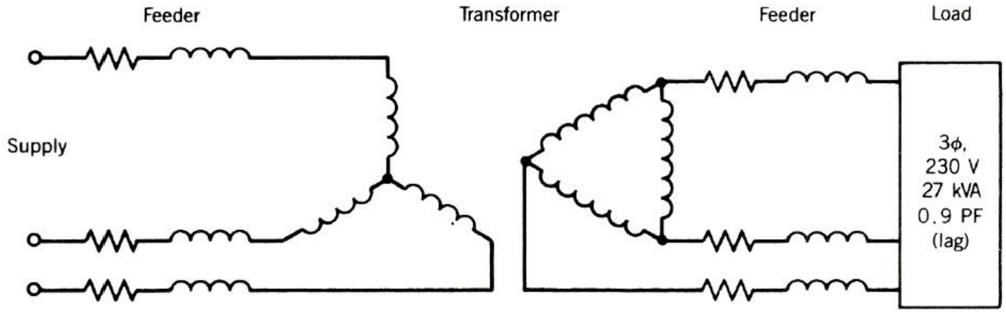
$$|V_1| = 2332.4\ \text{V}$$

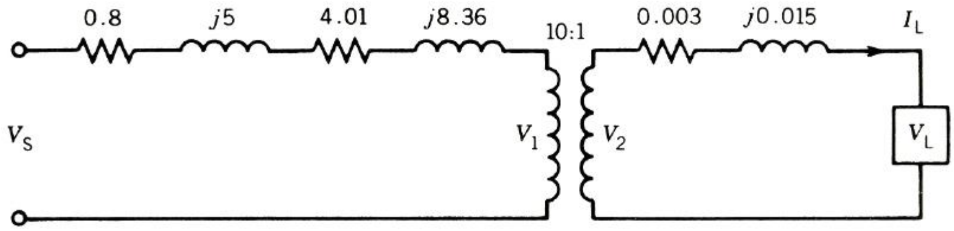
Primary line-to-line voltage $= \sqrt{3}V_1 = 4039.8\ \text{V}$

(d) $\text{VR} = \dfrac{2332.4 - 2300}{2300} \times 100\% = 1.41\%$

<u>Example</u>

A3-ph, 230 V, 27 kVA, 0.9 PF(lag) load is supplied by three 10 kVA, 1330/230V, 60 Hz transformers connected in Y-Δ by means of a common 3-ph feeder whose impedance is 0.003+j0.015 Ω per phase. The transformers are supplied from a 3-ph source through a 3-ph feeder whose impedance is 0.8+j5.0 Ω per phase. The equivalent impedance of one transformer referred to the low-voltage side is 0.12+j0.25 Ω. Determine the required supply voltage if the load voltage is 230 V.



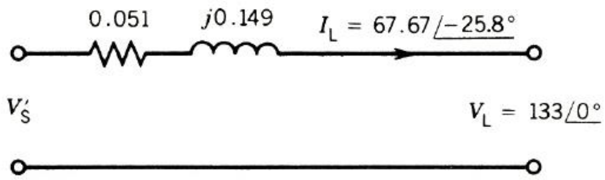


The equivalent circuit of the individual transformer referred to the high-voltage side is

$$\left(\frac{1330}{230}\right)^2(0.12+j0.25) = 4.01+j8.36$$

The turns ratio of the equivalent Y-Y bank is $\frac{\sqrt{3}\times 1330}{230} = 10$

The single-phase equivalent circuit of the system is shown below. All the impedances from the primary side can be transferred to the secondary side and combined with the feeder impedance on the secondary side.



$$R = (0.80+4.01)\frac{1}{10^2} + 0.003 = 0.051\ \Omega \qquad X = (5+8.36)\frac{1}{10^2} + 0.015 = 0.149\ \Omega$$

$$V_L = \frac{230}{\sqrt{3}}\angle 0^\circ = 133\angle 0^\circ\ \text{V} \qquad I_L = \frac{27\times 10^3}{3\times 133} = 67.67\ \text{A}$$

$$\phi_L = -\cos^{-1}0.9 = -25.8^\circ$$

$$V'_s = 133\angle 0^\circ + 67.67\angle{-25.8^\circ}(0.051+j0.149) = 133\angle 0^\circ + 10.6571\angle 45.3^\circ$$

$$= 140.7\angle 3.1^\circ\ \text{V}$$

$$V_s = 140.7\times 10 = 1407\ \text{V}$$

The line-to-line supply voltage is: $1407\sqrt{3} = 2437$ V

SHEET (2) Transformers

Problem (1):

A 400/200-V, 1-phase transformer is supplying a load of 25 A at a p.f. of 0.866 lagging. On no-load the current and power factor are 2 A and 0.208 respectively. Calculate the current taken from the supply. **[13.9 A lagging V_1 by 36.1°]**

Problem (2):

A transformer takes 10 A on no-load at a power factor of 0.1. The turns ratio is 4 : 1. If a load is supplied by the secondary at 200 A and p.f. of 0.8 lag, find the primary current and power factor. **[57.2 A; 0.717 lagging]**

Problem (3):

A transformer is supplied at 1600 V on the H.V. side and has a turns ratio of 8 : 1. The transformer supplies a load of 20 kW at a power factor of 0.8 lag and takes a magnetising current of 2.0 A at a power factor of 0.2. Calculate the magnitude and phase of the current taken from the H.V. supply. **[17.15 A ; 0.753 lag]**

Problem (4):

Draw the equivalent circuit for a 3000/400-V, 1-phase transformer on which the following test results were obtained. Input to high voltage winding when LV winding is open-circuited: 3000 V, 0.5 A, 500 W. Input to LV winding when HV winding is short-circuited: 11 V, 100 A, 500 W. **[R0 = 18,000 Ω, X0 = 6,360 Ω, R01 = 2.81 Ω, X01 = 5.51 Ω]**

Problem (5):

The ratio of turns of a 1-phase transformer is 8, the resistances of the primary and secondary windings are 0.85 Ω and 0.012 Ω respectively and leakage reactances of these windings are 4.8 Ω and 0.07 Ω respectively. Determine the voltage to be applied to the primary to obtain a current of 150 A in the secondary circuit when the secondary terminals are short-circuited. Ignore the magnetising current. **[176.4 W]**

Problem (6):

A 20 kVA, 2500/250 V, 50 Hz, 1-phase transformer has the following test results :

O.C. Test (LV side) : 250 V, 1.4 A, 105 W

S.C. Test (HV side) : 104 V, 8 A, 320 W

Compute the parameters of the approximate equivalent circuit referred to the low voltage side and draw the circuit. **(R0 = 592.5 Ω ; X0 = 187.2 Ω ; R02 = 1.25 Ω ; X12 = 3 Ω)**

Problem (7):

A 10-kVA, 2000/400-V, single-phase transformer has resistances and leakage reactances as follows :

$R1 = 5.2\ \Omega$, $X1 = 12.5\ \Omega$, $R2 = 0.2\ \Omega$, $X2 = 0.5\ \Omega$

Determine the value of secondary terminal voltage when the transformer is operating with rated primary voltage with the secondary current at its rated value with p. f. 0.8 lag. The no-load current can be neglected. Draw the phasor diagram. **[376.8 V]**

Problem (8):

A 50 kVA, 2200/110 V transformer when tested gave the following results :

O.C. test (L.V. side) : 400 W, 10 A, 110 V.

S.C. test (H.V. side) : 808 W, 20.5 A, 90 V.

Compute all the parameters of the equivalent circuit referred to the H.V. side.

[Shunt branch: Rc = 12100 Ω, Xm = 4723.36 Ω Series branch: r = 1.9224 Ω, x = 3.9467 Ω]

Problem (9):

A 200-kVA transformer has an efficiency of 98% at full-load. If the maximum efficiency occurs at three-quarters of full-load, calculate

(*a*) iron loss at F.L.

(*b*) Cu loss at F.L.

(*c*) efficiency at half-load.

Assume a p.f. of 0.8 at all loads. **[(*a*) 1.777 kW (*b*) 2.09 kW (*c*) 97.92%]**

Problem (10):

A 600 kVA, 1-ph transformer has an efficiency of 92 % both at full-load and half-load at unity power factor. Determine its efficiency at 60 % of full load at 0.8 power factor lag. **[90.59%]**

Problem (11):

Find the efficiency of a 150 kVA transformer at 25 % full load at 0.8 p.f. lag if the copper loss at full load is 1600 W and the iron loss is 1400 W. **[96.15%]**

Problem (12):

The F.L. Cu loss and iron loss of a transformer are 920W and 430W respectively.

(*i*) Calculate the loading of the transformer at which efficiency is maximum

(*ii*) what would be the losses for giving maximum efficiency at 0.85 of full-load if total full-load losses are to remain unchanged? **[(*i*) 68.4% of F.L. (*ii*) W*i* = 565 W ; W*cu* = 785 W]**

Problem (13):

At full-load, the Cu and iron losses in a 100-kVA transformer are each equal to 2.5 kW. Find the efficiency at a load of 65 kVA, power factor 0.8. **[93.58%]**

Problem (14):

A transformer, when tested on full-load, is found to have Cu loss 1.8% and reactance drop 3.8%. Calculate its full-load regulation (*i*) at unity p.f. (*ii*) 0.8 p.f. lagging (*iii*) 0.8 p.f. leading. **[(*i*) 1.80% (*ii*) 3.7 % (*iii*) −0.88%]**

Problem (15):

When a 100-kVA single-phase transformer was tested in this way, the following data were obtained: On open circuit, the power consumed was 1300 W and on short-circuit the power consumed was 1200 W. Calculate the efficiency of the transformer on (*a*) full-load (*b*) half-load when working at unity power factor. **[(*a*) 97.6% (*b*) 96.9%]**

Problem (16):

An 11,000/230-V, 150-kVA, 50-Hz, 1-phase transformer has a core loss of 1.4 kW and full-load Cu loss of 1.6 kW. Determine (*a*) the kVA load for maximum efficiency and the minimum efficiency (*b*) the efficiency at half full-load at 0.8 power factor lagging. **[140.33 kVA, 97.6% ; 97%]**

Problem (17):

A transformer working at unity p.f. has an efficiency of 90 % at both half-load and a full-load of 500 kW. Determine the efficiency at 75 % of full-load **[90.5028%]**

Problem (18):

A 10-kVA, 500/250-V, single-phase transformer has its maximum efficiency of 94 % when delivering 90 % of its rated output at unity power factor. Estimate its efficiency when delivering its full-load output at p.f. of 0.8 lagging. **[92.6%]**

Problem (19):

A single-phase transformer has a voltage ratio on open-circuit of 3300/660-V. The primary and secondary resistances are 0.8 Ω and 0.03 Ω respectively, the corresponding leakage reactance being 4 Ω and 0.12 Ω. The load is equivalent to a coil of resistance 4.8 Ω and inductive reactance 3.6 Ω. Determine the terminal voltage of the transformer and the output in kW. **[636 V, 54 kW]**

Problem (20):

A 100-kVA, single-phase transformer has an iron loss of 600 W and a copper loss of 1.5 kW at full-load current. Calculate the efficiency at (*a*) 100 kVA output at 0.8 p.f. lagging (*b*) 50 kVA output at unity power factor. **[(*a*) 97.44% (*b*) 98.09%]**

Problem (21):

A 10-kVA, 440/3300-V, 1-phase transformer, when tested on open circuit, gave the following figures on the primary side : 440V ; 1.3 A ; 115 W.

When tested on short-circuit with full-load current flowing, the power input was 140 W. Calculate the efficiency of the transformer at (*a*) full-load unity p.f. (*b*) one quarter full-load 0.8 p.f. **[(*a*) 97.51% (*b*) 94.18%]**

Problem (22):

A 100-kVA distribution transformer has a maximum efficiency of 98 % at 50 % full-load and unity power factor. Determine its iron losses and full-load copper losses.

The transformer undergoes a daily load cycle as follows:

Load	*Power factor*	*Load duration*
100 kVA	1.0	8 hrs
50 kVA	0.8	6 hrs
No load		10 hrs

Determine all-day efficiency. **[$P_{cu\ FL}$=2040.8163 W, P_{iron}=510.2041 W, $\eta_{all\ day}$=97.05%]**

Problem (23):

A 20-kVA transformer has a maximum efficiency of 98 % when delivering three-fourth full-load at u.p.f. If during the day, the transformer is loaded as follows:

12 hours No load; 6 hours 12 kWh, 0.8 p.f.; 6 hours 20 kW, u.p.f.

Calculate the energy efficiency of the transformer.